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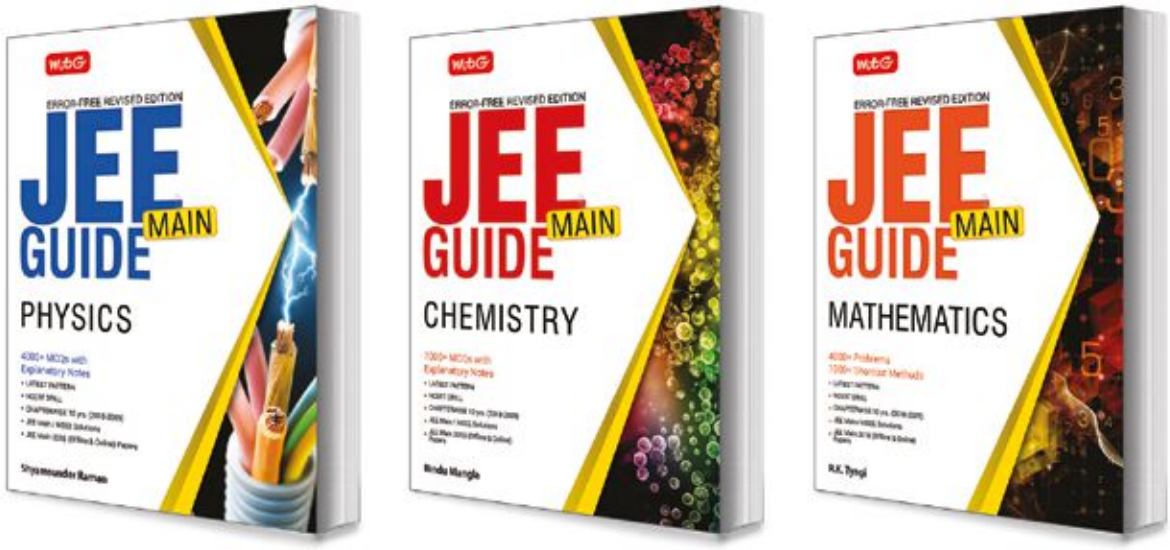

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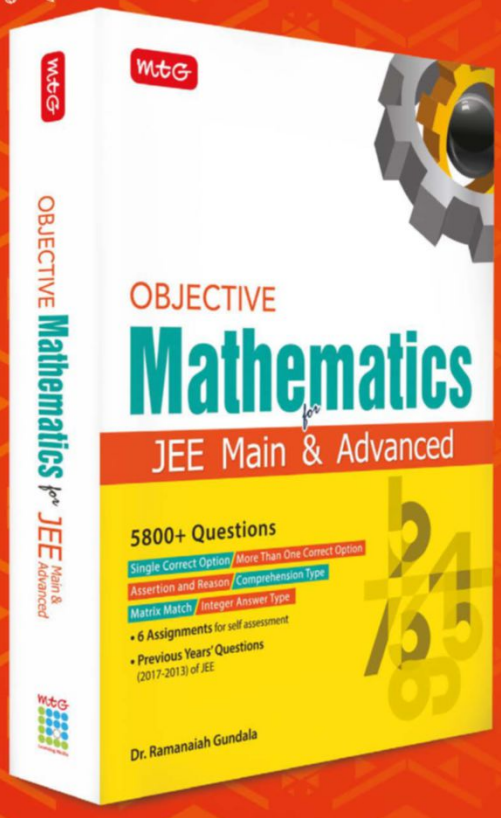

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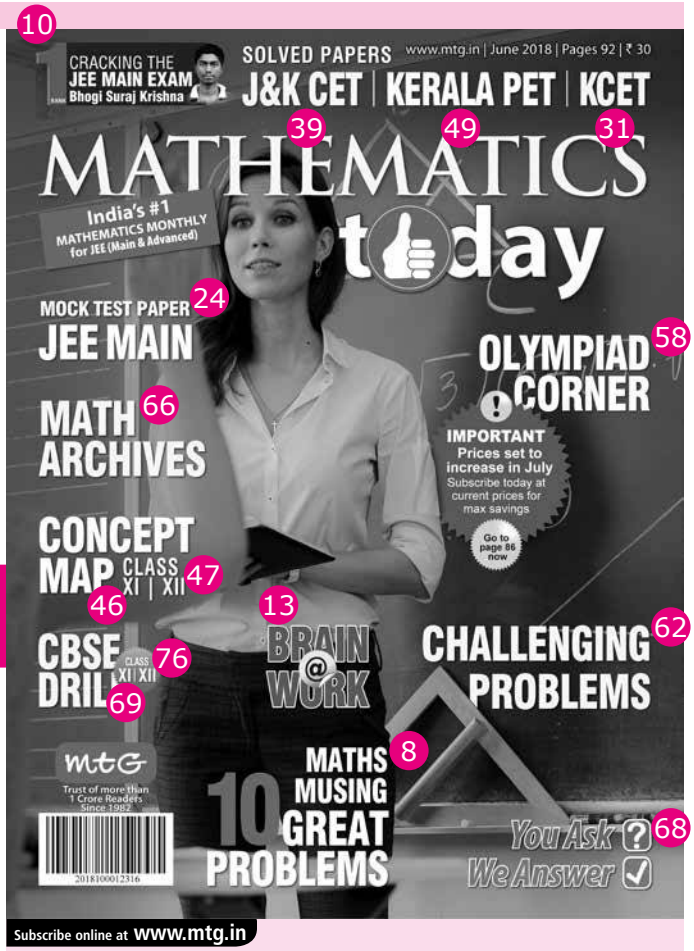

TO OUR READERS

We are happy that intelligent students, teachers and other professionals continue to patronise Mathematics Today, Chemistry Today, Physics For You and Biology Today.

To them, we are addressing this open letter in view of increase in the cost of production and postage in the last seven years. All round spiralling prices have pushed production costs so high, that many in out fraternity find it impossible to continue business. We are compelled to raise the price to ₹ 40 from July 2018 issue.

We understand the pressure of cost on the student-teacher community in general but, we are hoping our readers will understand our problems and that we have no option but to comply with this unavoidable move.

We on our part, will keep up our efforts to improve the magazines in all its aspects.

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.
During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.
Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 186

JEE MAIN

1. If α, β be the roots of $m^2(x^2 - x) + 2mx + 3 = 0$ and m_1, m_2 be the two values of m for which α, β are connected by the relation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{3}$ the value of $\frac{m_1^2}{m_2} + \frac{m_2^2}{m_1}$ is

(a) $\frac{-58}{3}$ (b) $\frac{-68}{3}$ (c) $\frac{-76}{3}$ (d) $\frac{-86}{3}$.

2. Let $f(x) = \begin{cases} |x^2 - 1| - 1 & , x \le 1 \\ |2x - 3| - |x - 2), & x > 1 \end{cases}$. The number of points at which $f(x)$ is not continuous is

(a) 0 (b) 1 (c) 2 (d) More than 2

3. Which of the following is the solution of D.E. $xy^2 \frac{dy}{dx} = 1 - x^2 + y^2 - x^2 y^2$?

(a) $y + 2\tan^{-1} y = \ln x + \frac{x^2}{3} + C$

(b) $y - \frac{1}{2}\tan^{-1} y = \ln x + \frac{x^2}{2} + C$

(c) $y - \tan^{-1} y = \ln x - \frac{x^2}{2} + C$

(d) None of these

4. For a student to qualify he must pass at least two out of three examinations. The probability that he will pass the first examination is p. If he fails in one of the examination, then the probability of his passing in the next examination is $p/2$, otherwise it remains the same. Find the probability that he will qualify.

(a) $2p^3 - p^2$ (b) $2p^2 - p^3$
(c) $p^3 - 2p^2$ (d) $p^2 - 2p^3$

5. In which of the following intervals can x lie so that it satisfies the inequality $\frac{2x}{2x^2 + 5x + 2} > \frac{1}{x + 1}$?

(a) $(-2, -1)$ (b) $\left(-\frac{2}{3}, \frac{1}{2}\right)$

(c) $\left(-\frac{1}{3}, \frac{1}{2}\right)$ (d) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$

JEE ADVANCED

6. Let $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$. Find the value of $\cos^2\alpha + \cos^2\alpha + \cos^2\gamma$.

(a) 1/2 (b) 1 (c) 3/2 (d) 2

COMPREHENSION

Consider the n numbers : $n + 1$, $n + 2$, $n + 3$, ..., $n + n$. $n \in N$. The A.M., G.M., H.M. and R.M.S. (Root-mean square) of the numbers are A_n, G_n, H_n, R_n respectively. Then

7. $\lim_{n\to\infty} \frac{G_n}{n} =$

(a) $1/e$ (b) $2/e$ (c) $3/e$ (d) $4/e$

8. $\lim_{n\to\infty} \frac{R_n}{n} =$

(a) $\sqrt{3}$ (b) 2 (c) $\sqrt{\frac{7}{3}}$ (d) 3

INTEGER MATCH

9. Let $\vec{a} = 2\hat{i} + \hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $c = 4\hat{i} - 3\hat{j} + 7\hat{k}$. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be a vetcor such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$. The magnitude of $x + y + z$ is

(a) 3 (b) 5 (c) 7 (d) 9

MATRIX MATCH

10. Match the following.

	List-I	List-II	
P.	Number of ordered pairs which satisfy the equation $x^2 + 2x\sin(xy) + 1 = 0$ (where $y \in [0, 2\pi]$) is	1.	1
Q.	If maximum $\{5\sin\theta + 3\sin(\theta - \alpha)\} = 7(\theta \in R)$ then number of solutions of α in $[0, \pi]$ are	2.	2
R.	The total number of solutions of $\cos x = \sqrt{1 - \sin 2x}$ in $[0, 2\pi]$ is equal to	3.	0
S.	The number of solutions of the equation $\sqrt{\sin x} + 2^{1/4}\cos x = 0$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ are	4.	3

	P	Q	R	S
(a)	1	4	2	3
(b)	2	1	2	3
(c)	1	2	1	4
(d)	3	2	1	4

See Solution Set of Maths Musing 185 on page no. 65

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Cracking the JEE Main EXAM

Bhogi Suraj Krishna

JEE Main 2018 Topper Interview: Bhogi Suraj Krishna (AIR 1) - "Passion for your goals is a driving force"

Bhogi Suraj Krishna was inundated with congratulatory messages on April 30. Topping JEE Main 2018 with an All India Rank of 1 was the icing on the cake for the Andhra boy for what has been a two year journey filled with hard work and supported by passion. Suraj Krishna was so focussed on his studies that he gave up his passion for movies during this period. He feels that his passion for his studies was the driving force behind his success. Here he talks on why he chose to write the JEE Main offline exam, importance of studying NCERT text books, solving past year question papers and finally, liking what one chooses a career path in. Read the complete interview given here to delve into the success mantras of a JEE Main topper.

Congratulations for your outstanding performance in JEE Main 2018! What was your reaction upon knowing your rank?

Bhogi Suraj Krishna: Once I got to know the result of JEE Main 2018, I was very happy and excited. I secured the first rank; the feeling is still sinking in.

Tell us something about yourself. Which board have you studied under? Which school have you studied in?

Bhogi Suraj Krishna: I am from Andhra Pradesh. I had studied my qualifying exam from the Andhra Board. My schooling for Intermediate was from Sri Chaitanya.

What is your score in JEE Main 2018? What is your all India rank and category rank?

Bhogi Suraj Krishna: I scored 350 out of 360 in JEE Main 2018. I secured an AIR Rank 1 and my category rank is also the same.

Did you appear for the offline JEE Main exam or the online one? What is the reason behind this choice of yours?

Bhogi Suraj Krishna: I took the exam in offline mode. I am not very comfortable with the online mode of exam hence my choice. See, one should choose what one is comfortable with as it matters a lot in an exam of this stature.

Why do you want to study engineering? When did you decide to study engineering? Which branch would you be interested in? Any reason?

Bhogi Suraj Krishna: I have always been interested in engineering. I was in Class IX when I made up my mind to take up engineering as a career. I would like to pursue computer engineering and one of the primary reasons for this is that I am deeply interested in the field.

When did you start your JEE Main preparation? Tell us something

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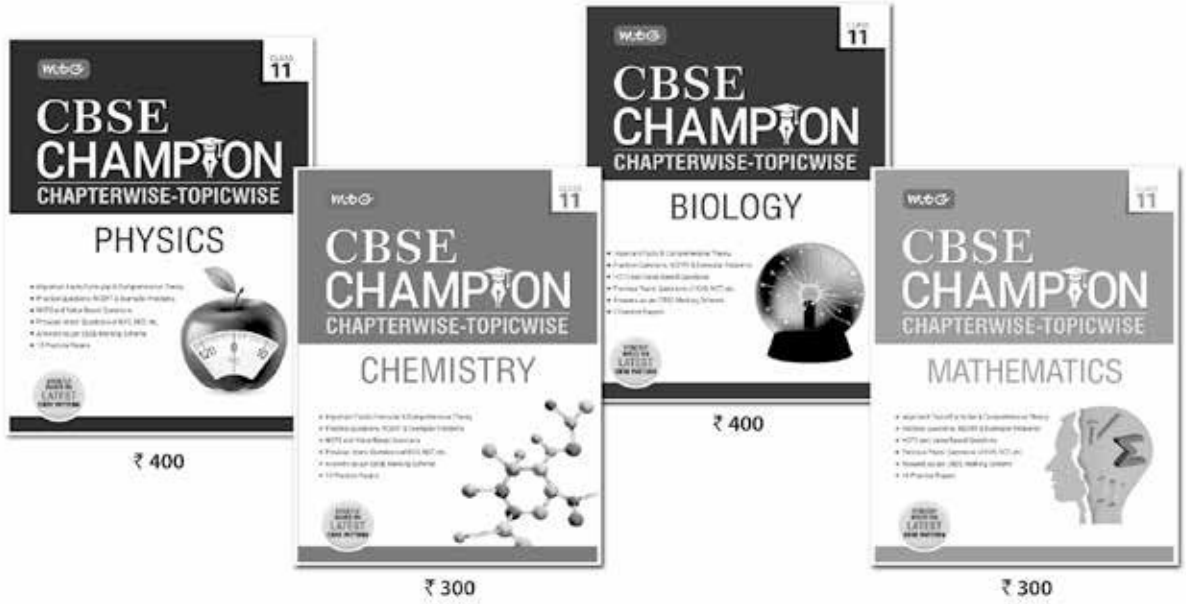

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about your preparation strategy and daily routine for JEE Main?

Bhogi Suraj Krishna: I started preparing for JEE Main from class IX. There is a foundation course for cracking the JEE exam. I had registered for it when I was in class IX. During the course of preparation for JEE Main 2018, I always approached my faculty when I ran into difficulties or had queries. I sought their assistance in clearing up all of those doubts and would not give up till my concepts were clear. I used to study for about nine hours every day.

Was there a difference in the preparation strategy in the last months of JEE Main considering that the board exams also are during this period? How did you manage preparation for both?

Bhogi Suraj Krishna: I did not prepare much for the board examination. Since the syllabus for Physics, Chemistry and Mathematics is the same for the board exam and JEE Main 2018, my main focus was on JEE Main as well as on JEE Advanced. I seriously did not face any problems as I was prepared. The only difference in both the exams is that one is subjective while the other one is objective. If your concepts are clear, you can tackle them all.

Did you take coaching for JEE Main and if yes from where? What are the benefits of coaching according to you?

Bhogi Suraj Krishna: I took coaching at Sri Chaitanya. One of the biggest advantages of taking coaching is that they help you devise the right strategy to crack the exam. It also depends upon you to study and put that to use. One should never give up till one is clear about the concepts. Also if you follow what they devise, then you can easily score good marks.

Mock tests and sample papers – how extensively did you practise with the help of these? Do you think practising from the previous year papers help and how?

Bhogi Suraj Krishna: I practised extensively from JEE Main sample papers. It is important to practise from the past years sample papers of JEE Main as it helps to know the level and type of questions that are asked. You are prepared for the exam in all ways if you use the past year papers for preparation. This is a must for all aspirants. You see, you can gauge your levels of preparation and hone your time management skills. If needed you could rectify and proceed with your preparation.

Are there any special books that you feel have helped with your JEE Main preparation? Are the school text books adequate for preparing for JEE Main and Class XII?

Bhogi Suraj Krishna: While preparing for JEE Main 2018, I studied mainly from the NCERT books and from the notes that were provided by the coaching centre. This is more important for Chemistry where one needs to study from the NCERT books and faculty notes. As far as Physics and Mathematics are concerned, it is important to solve a lot of questions in addition to the regular preparation form the study notes and books.

Did you face any problem during the JEE Main examination? Which subject was the easiest and which was the toughest in JEE Main? What do you think is a tough thing to manage during the JEE Main exam?

Bhogi Suraj Krishna: I did not face any problem during the JEE Main 2018 exam. Well, I did feel that there were some ambiguous questions in Chemistry. Mathematics and Physics – I felt were very lengthy. Time management is one of the most challenging factor in the JEE Main exam. If you can manage that, then it's good going for you.

"My message to all those students who are aspiring to take up JEE Main is that - Be comfortable with the subjects. Enjoy learning and enjoy the preparation time you spend. Be passionate about your goals as you can drive yourself to achieve them only if you like them."

What do you think are the key factors behind your stupendous success in JEE Main?

Bhogi Suraj Krishna: I would like to attribute my success to the faculty of Sri Chaitanya as they have been very supportive and provided a lot of motivation.

How did you de-stress yourself during the preparation? What are your hobbies? How often could you pursue them?

Bhogi Suraj Krishna: Every fortnight, we were taken out and that was very relaxing. Right amount of sleep is important. For me that was a de-stressing element. My hobbies include watching movies. However, during the entire duration of my preparation, I did give it up as my goals were more important.

You know you are an inspiration to thousands of students across the country. Do you have any message for future aspirants on tackling the JEE Main exam?

Bhogi Suraj Krishna: My message to all those students who are aspiring to take up JEE Main is that - Be comfortable with the subjects. Enjoy learning and enjoy the preparation time you spend. Be passionate about your goals as you can drive yourself to achieve them only if you like them.

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BRAIN @ WORK

QUADRATIC EQUATIONS

IDENTITY

An identity is the statement of equality between two expressions which is always true for all values of the variables involved. So, $f(x) = g(x)$ is an identity if $f(x)$ and $g(x)$ have same value for every value of x.

Note:

(i) A polynomial of degree n represents an identity, if it is satisfied by $(n + 1)$ or more values of x.

(ii) If $f(x) = g(x)$ represents an identity, then the coefficients of similar terms of x are equal.

(iii) An equation $ax^3 + bx^2 + cx + d = 0$ represents an identity in terms of x, then $a = b = c = d = 0$.

PROPERTIES OF ROOTS OF EQUATION

Factor theorem

(i) $(x - \alpha)$ is a factor of a polynomial $f(x)$ if and only if $f(\alpha) = 0$.

(ii) $(x - \alpha)^2$ is a factor of a polynomial $f(x)$ if and only if $f(\alpha) = f'(\alpha) = 0$. In this case, we say that α is a repeated root of $f(x) = 0$ (a double root).

(iii) If $(x - \alpha)^m$ is a factor of a polynomial $f(x) = 0$, then $f(\alpha) = f'(\alpha) = f''(\alpha) = f'''(\alpha) = ... = f^{(m-1)}(\alpha) = 0$ and $f^m(\alpha) \neq 0$.

Remainder theorem

The value of remainder, when $f(x)$ is divided by $(x - \alpha)(x - \beta)$, is

$$\left(\frac{f(\alpha)-f(\beta)}{\alpha-\beta}\right)x+\left(\frac{af(\beta)-\beta f(\alpha)}{\alpha-\beta}\right).$$

Position of roots of a polynomial equation

If $f(x) = 0$ is an equation and a, b are two real numbers such that

(i) $f(a)\, f(b) < 0$, then the equation $f(x) = 0$ has at least one real root or an odd number of real roots between a and b.

(ii) If $f(a)$ and $f(b)$ are of the same sign, then either no real root or an even number of real roots of $f(x) = 0$ lie between a and b.

RELATION BETWEEN ROOTS AND CO-EFFICIENTS OF POLYNOMIAL EQUATION

Consider the general equation of n^{th} degree

$f(x) = a_0 x^n + a_1 x^{n-1} + ... + a_{n-1} x + a_n = 0$, where $a_0, a_1, ..., a_n \in C$ and $n \in Z$

Let its roots be $\alpha_1, \alpha_2 ...., \alpha_n$. Then,

- Sum of roots taken one at a time $= S_1 = \sum \alpha_i = -\frac{a_1}{a_0}$

  Sum of product of roots taken two at a time $= S_2$

  $$= \sum_{i \neq j} \alpha_i \alpha_j = \frac{a_2}{a_0}$$

  Sum of product of roots taken three at a time $= S_3$

  $$= \sum_{i \neq j \neq k} \alpha_i \alpha_j \alpha_k = -\frac{a_3}{a_0}$$

  .....

  .....

  S_n = the product of roots taken all at a time $= S_n$

  $$= \alpha_1 \alpha_2 .... \alpha_n = (-1)^n \cdot \frac{a_n}{a_0}$$

  Number of terms in $S_1, S_2, S_3, ..., S_n$ are respectively ${}^nC_1, {}^nC_2, {}^nC_3, ..., {}^nC_n$.

- $f(\alpha_1) = f(\alpha_2) = f(\alpha_3) = f(\alpha_4) = ... = f(\alpha_n) = 0$
- If $f(x) = 0$ has n real roots, then $f'(x) = 0$ has $(n - 1)$ real roots.
- If $f(x) = 0$ has n real roots, then $f(x) = a_0(x - \alpha_1)(x - \alpha_2)(x - \alpha_3) .... (x - \alpha_{n-1})(x - \alpha_n)$
- If $\alpha_1, \alpha_2, \alpha_3 ....., \alpha_n$ are n roots of an equation, then the equation can be written as $x^n - S_1 x^{n-1} + S_2 x^{n-2} + ... + (-1)^n S_n = 0$.

TRANSFORMATION OF EQUATIONS

- An equation whose roots are reciprocals of the roots of a given equation is obtained by replacing x by $1/x$ in the given equation and simplify it to make it a polynomial equation.
- An equation whose roots are negative of the roots of a given equation is obtained by replacing x by $-x$ in the given equation and simplify it to make it a polynomial equation.
- An equation whose roots are squares of the roots of a given equation is obtained by replacing x by $\sqrt{x}$ in the given equation and simplify it to make it a polynomial equation.
- An equation whose roots are cubes of the roots of a given equation is obtained by replacing x by $x^{1/3}$ in the given equation and simplify it to make it a polynomial equation.

QUADRATIC EQUATION

An equation that can be written in the form $ax^2 + bx + c = 0 \ \forall \ \ a, b, c \in R$ and $a \neq 0$, is called a quadratic equation.

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

The quantity $b^2 - 4ac$ is called the discriminant of the quadratic equation and is denoted by D or Δ.

Usually, the two roots of $ax^2 + bx + c = 0$ are denoted by α and β. The expression $ax^2 + bx + c$ can thus be written as $ax^2 + bx + c = a(x - \alpha)(x - \beta)$.

Sum and Product of Roots

$$\text{Sum of roots} = S = \alpha + \beta = \frac{-b}{a} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

$$\text{Product of roots} = P = \alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Also, $(\text{Difference of roots})^2 = (\alpha - \beta)^2$

$$= (\alpha + \beta)^2 - 4\alpha\beta = \frac{b^2 - 4ac}{a^2} \Rightarrow |\alpha - \beta| = \frac{\sqrt{D}}{|a|}$$

The quadratic equation with sum of roots S and product of roots P is given by $x^2 - Sx + P = 0$

i.e., $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

The nature of the roots of $ax^2 + bx + c = 0$

(i) The roots are real and distinct iff $D > 0$.

(ii) The roots are real and equal iff $D = 0$ and the equal root is given by $x = -b/2a$.

When $D = 0$, $ax^2 + bx + c$ is a perfect square. Under this condition $ax^2 + bx + c = \left\{\sqrt{a}\left(x + \frac{b}{2a}\right)\right\}^2$

(iii) The roots are complex with non-zero imaginary part iff $D < 0$.

(iv) The roots are rational iff a, b, c are rational and D is a perfect square.

(v) The roots are of the form $p + \sqrt{q}$ $(p, q \in Q)$ *i.e.*, irrational iff a, b, c are rational and D is not a perfect square.

(vi) If a quadratic equation in x has more than two roots, then it is an identity in x.

Nature of the Roots of $f(x)\cdot g(x) = 0$

If D_1 and D_2 are the discriminants of the quadratic equations $f(x) = 0$ and $g(x) = 0$, then the following possibilities arises about the roots of the equation $f(x)\cdot g(x) = 0$ are

(i) If $D_1 + D_2 \geq 0$, then there will be at least two real roots of the equation $f(x) \cdot g(x) = 0$.

(ii) If $D_1 + D_2 < 0$, then there will be at least two imaginary roots of $f(x) \cdot g(x) = 0$.

(iii) If $D_1 \cdot D_2 < 0$, then the equation $f(x)\cdot g(x) = 0$ will have two real roots.

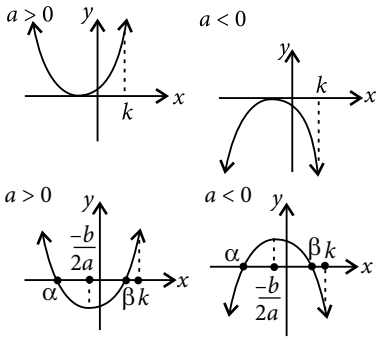
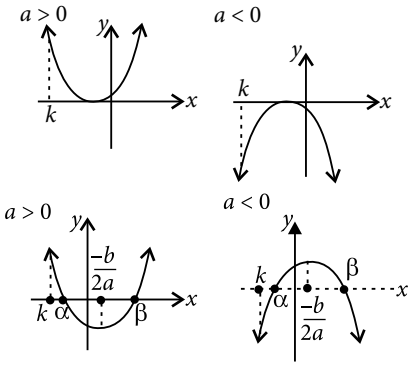
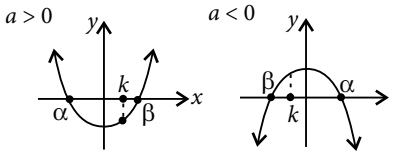
(iv) If $D_1 D_2 > 0$ then the equation $f(x)\cdot g(x) = 0$ has either four real roots or no real root.

CONDITION FOR COMMON ROOTS

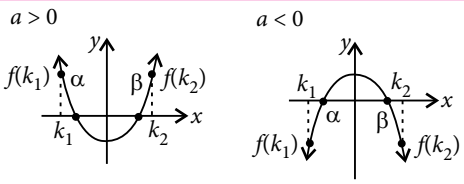
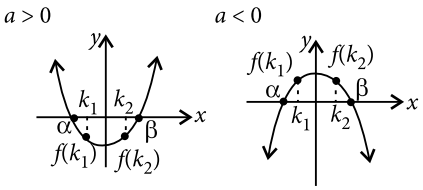
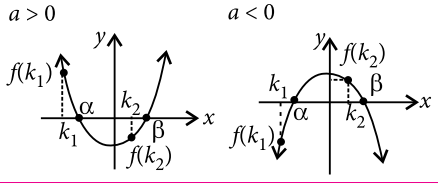
- If $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ have one common root, then $(a_1b_2 - a_2b_1)(b_1c_2 - c_1b_2) = (c_1a_2 - a_1c_2)^2$
- If $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ have both roots common, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- If the two equation $a_1x^2 + b_1x + c_1 = 0$, $a_2x^2 + b_2x + c_2 = 0$ with real coefficients have an imaginary root common, then both roots will be common, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- If the two equation $a_1x^2 + b_1x + c_1 = 0$; $a_2x^2 + b_2x + c_2 = 0$ with rational coefficients have an irrational root common, then both roots will be common, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- If every pair of three quadratic equations have a common root, then roots are taken as $\alpha, \beta; \beta, \gamma; \gamma, \alpha$
- If a quadratic equation and cubic equation have a common root, try to find the root of cubic equation by factorization.

POSITION OF ROOTS OF THE QUADRATIC EQUATION $ax^2 + bx + c = 0$

With Respect to One Quantity (k)

S.No.	Situation	Graphical Representation	Required Conditions
1.	Both the roots are less than k *i.e.*, $\alpha < \beta < k$		(i) $D \geq 0$ (ii) $a\,f(k) > 0$ (iii) $k > \frac{-b}{2a}$
2.	Both the roots are greater than k *i.e.*, $k < \alpha < \beta$		(i) $D \geq 0$ (ii) $a\,f(k) > 0$ (iii) $k < \frac{-b}{2a}$
3.	k lies between the roots *i.e.*, $\alpha < k < \beta$		(i) $D > 0$ (ii) $af(k) < 0$

With Respect to Two Quantities k_1 and k_2

S.No.	Situation	Graphical Representation	Required Conditions
1.	Distinct roots lies in the interval (k_1, k_2) *i.e.*, $k_1 < \alpha < \beta < k_2$		(i) $D > 0$ (ii) $a\,f(k_1) > 0$ (iii) $af(k_2) > 0$ (iv) $k_1 < \frac{-b}{2a} < k_2$
2.	Interval (k_1, k_2) lies between the roots *i.e.*, $\alpha < k_1 < k_2 < \beta$		(i) $D > 0$ (ii) $a\,f(k_1) < 0$ (iii) $af(k_2) < 0$
3.	One root lies in the interval (k_1, k_2) *i.e.*, $k_1 < \alpha < k_2 < \beta$		(i) $D > 0$ (ii) $f(k_1)\,f(k_2) < 0$

PROBLEMS

1. The natural number n for which the expression $y = 5(\log_3 n)^2 - \log_3 n^{12} + 9$, has the minimum value is
(a) 2 (b) 3 (c) $3^{6/5}$ (d) 4

2. If α, β are the roots of the quadratic equation $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$, then $(\alpha - \gamma)(\alpha - \delta)$ is equal to
(a) $q + r$ (b) $q - r$
(c) $-(q + r)$ (d) $-(p + q + r)$

3. If α and β be the roots of the equation $x^2 + 3x + 1 = 0$, then the value of $\left(\frac{\alpha}{1+\beta}\right)^2 + \left(\frac{\beta}{\alpha+1}\right)^2$ is equal to
(a) 15 (b) 18 (c) 21 (d) none

4. The roots of the equation $x^2 + 6x + a = 0$ are real and distinct and they differ by atmost 4, then the range of value of a, is
(a) (5, 9] (b) [5, 9) (c) [4, 8) (d) [3, 9)

5. If the equation $\cot^4 x - 2\,\text{cosec}^2 x + a^2 = 0$ has atleast one solution then, sum of all possible integral values of 'a' is equal to
(a) 4 (b) 3 (c) 2 (d) 0

6. If the equation $4x^2 - 4(5x + 1) + p^2 = 0$ has one root equals to two more than the other, then the value of p is equal to
(a) $\pm\frac{\sqrt{236}}{3}$ (b) ± 5
(c) 5 or -1 (d) 4 or -3

7. The values of k for which the quadratic equation $(1 - 2k)x^2 - 6kx - 1 = 0$ and $kx^2 - x + 1 = 0$ have atleast one root in common are
(a) $\left\{\frac{1}{2}\right\}$ (b) $\left\{\frac{1}{3}, \frac{2}{9}\right\}$ (c) $\left\{\frac{2}{9}\right\}$ (d) $\left\{\frac{1}{2}, \frac{2}{9}\right\}$

8. The minimum value of the expression $|x - p| + |x - 15| + |x - p - 15|$ for 'x' in the range $p \le x \le 15$ where $0 < p < 15$, is
(a) 10 (b) 15 (c) 30 (d) 0

9. If x, y, z are real such that $x + y + z = 4, x^2 + y^2 + z^2 = 6$, then the range of x is
(a) $(-1, 1)$ (b) $[0, 2]$
(c) $[2, 3]$ (d) $[2/3, 2]$

10. The roots of the equation $a(x - b)(x - c) + b(x - c)(x - a) + c(x - a)(x - b) = 0$($a, b, c$ are distinct and real) are always:
(a) positive (b) negative
(c) real (d) unreal

11. If $\tan\theta$ and $\cot\theta$ are the roots of the equation $x^2 + 2x + 1 = 0$, then the least value of $x^2 + \tan\theta x + \cot\theta = 0$, is
(a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $\frac{-5}{4}$ (d) $\frac{-3}{4}$

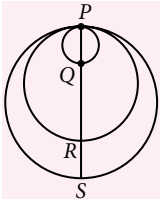
12. If one solution of the equation $x^3 - 2x^2 + ax + 10 = 0$ is the additive inverse of another, then which one of the following inequalities is true?
(a) $-40 < a < -30$ (b) $-30 < a < -20$
(c) $-20 < a < -10$ (d) $-10 < a < 0$

13. Suppose a, b and c are positive numbers such that $a + b + c = 1$, then the maximum value of $ab + bc + ca$ is
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

14. Assume that p is a real number. In order for $\sqrt[3]{x+3p+1} - \sqrt[3]{x} = 1$ to have real solutions, it is necessary that
(a) $p \ge 1/4$ (b) $p \ge -1/4$
(c) $p \ge 1/3$ (d) $p \ge -1/3$

15 Let α, β, γ are roots of the equation $x^3 + qx + q = 0$, then find the value of $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1}$.
(a) 0 (b) -1 (c) 1 (d) none

16. $PQRS$ is a common diameter of three circles. The area of the middle circle is the average of the area of the other two. If $PQ = 2$ and $RS = 1$ then the length QR is



(a) $\sqrt{6}+1$ (b) $\sqrt{6}-1$
(c) 5 (d) 4

17. If every solution of the equation $3\cos^2 x - \cos x - 1 = 0$ is a solution of the equation $a\cos^2 2x + b\cos 2x - 1 = 0$. Then the value of $(a + b)$ is equal to
(a) 5 (b) 9 (c) 13 (d) 14

18. If all values of x obtained from the equation $4^x + (k - 3)2^x + k = 4$ are non-positive, then the largest integral value of k is
(a) 1 (b) 2 (c) 3 (d) 4

19. Let r_1, r_2 and r_3 be the solutions of the equation $x^3 - 2x^2 + 4x + 5074 = 0$ then the value of $(r_1 + 2)(r_2 + 2)(r_3 + 2)$ is
(a) 5050 (b) 5066 (c) -5050 (d) -5066

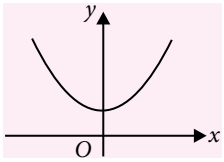
20. If α and β are the roots of the equation $(\log_2 x)^2 + 4(\log_2 x) - 1 = 0$, then the value of $\log_\beta \alpha + \log_\alpha \beta$ equals

(a) 18 (b) −16 (c) 14 (d) −18

21. Let $m(b)$ be the minimum value of $f(x) = (2 + b + b^2)x^2 - 2\sqrt{2}\,(2b + 1)x + 8$, where $b \in [-3, 10]$. The maximum value of $m(b)$ is

(a) 2 (b) 4 (c) 6 (d) 8

22. The graph of a quadratic polynomial $y = ax^2 + bx + c$ $(a, b, c \in R)$ with vertex on y-axis is as shown in the figure.



Then which one of the following statement is INCORRECT?

(a) Product of the roots of the corresponding quadratic equation is positive.
(b) Discriminant of the quadratic equation is negative.
(c) Both (a) and (b)
(d) None of these

23. The quadratic equation $x^2 - 1088x + 295680 = 0$ has two positive integral roots whose greatest common divisor is 16. The least common multiple of the two roots is

(a) 18240 (b) 18480
(c) 18960 (d) 19240

24. Given a, b, c are non negative real numbers and if $a^2 + b^2 + c^2 = 1$, then the value of $a + b + c$ is

(a) ≥ 3 (b) ≥ 2 (c) $\leq \sqrt{2}$ (d) $\leq \sqrt{3}$

25. The set of values of 'a' for which the inequality, $(x - 3a)(x - a - 3) < 0$ is satisfied for all $x \in [1, 3]$ is

(a) $(1/3, 3)$ (b) $(0, 1/3)$
(c) $(-2, 0)$ (d) $(-2, 3)$

26. If the roots of the cubic, $x^3 + ax^2 + bx + c = 0$ are three consecutive positive integers. Then the value of $\dfrac{a^2}{b+1}$ is equal to

(a) 3 (b) 2 (c) 1 (d) 1/3

27. If x be the real number such that $x^3 + 4x = 8$, then the value of the expression $x^7 + 64x^2$ is

(a) 124 (b) 125 (c) 128 (d) 132

28. If the roots of the equation $x^3 - px^2 - r = 0$ are $\tan\alpha$, $\tan\beta$ and $\tan\gamma$, then the value of $\sec^2\alpha \cdot \sec^2\beta \cdot \sec^2\gamma$ is

(a) $p^2 + r^2 + 2rp + 1$ (b) $p^2 + r^2 - 2rp + 1$
(c) $p^2 - r^2 - 2rp + 1$ (d) None

29. Number of quadratic equations with real roots which remain unchanged even after squaring their roots, is

(a) 1 (b) 2 (c) 3 (d) 4

30. For a, b, c non-zero, real distinct, the equation, $(a^2 + b^2)x^2 - 2b(a + c)x + b^2 + c^2 = 0$ has non-zero real roots. One of these roots is also the root of the equation

(a) $a^2x^2 - a(b - c)x + bc = 0$
(b) $a^2x^2 + a(c - b)x - bc = 0$
(c) $(b^2 + c^2)x^2 - 2a(b + c)x + a^2 = 0$
(d) $(b^2 - c^2)x^2 + 2a(b - c)x - a^2 = 0$

31. If roots of the quadratic equation $x^2 + c = bx$ are two consecutive integers, then $b^2 - 4c$ equals

(a) −1 (b) 2 (c) 1 (d) 0

32. If the equation $\sin^4 x - (k + 2)\sin^2 x - (k + 3) = 0$ has a solution, then k must lie in the interval

(a) $(-4, -2)$ (b) $[-3, 2)$
(c) $(-4, -3)$ (d) $[-3, -2]$

33. Number of solutions of the equation $\sqrt{x^2} - \sqrt{(x-1)^2} + \sqrt{(x-2)^2} = \sqrt{5}$, is

(a) 0 (b) 1
(c) 2 (d) More than 2

34. Let a, b, c be three real numbers such that $a + b + c = 0$ and $a^2 + b^2 + c^2 = 2$. Then the value of $(a^4 + b^4 + c^4)$ is equal to

(a) 2 (b) 5 (c) 6 (d) 8

35. A solution of the equation $4^x + 4 \cdot 6^x = 5.9^x$ is

(a) −1 (b) 1 (c) 2 (d) 0

36. Consider two quadratic expressions $f(x) = ax^2 + bx + c$ and $g(x) = ax^2 + px + q$, $(a, b, c, p, q \in R, b \neq p)$ such that their discriminants are equal. If $f(x) = g(x)$ has a root $x = \alpha$, then

(a) α will be A.M. of the roots of $f(x) = 0$ and $g(x) = 0$
(b) α will be A.M. of the roots of $f(x) = 0$
(c) α will be A.M. of the roots of $f(x) = 0$ or $g(x) = 0$
(d) α will be A.M. of the roots of $g(x) = 0$

37. Consider the two functions $f(x) = x^2 + 2bx + 1$ and $g(x) = 2a(x + b)$, where the variable x and the constants a and b are real numbers. Each such pair of the constants a and b may be considered as a point (a, b) in an ab – plane. Let S be the set of such points (a, b) for which the graphs of $y = f(x)$ and $y = g(x)$ do not intersect (in the xy – plane.). The area of S is

(a) 1 (b) π (c) 4 (d) 4π

38. The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the mean of its zeroes, the product of its zeroes, and the sum of its coefficients are all equal. If the y-intercept of the graph of $y = P(x)$ is 2, then the value of b is

(a) -11 (b) -9 (c) -7 (d) 5

39. A quadratic equation, product of whose roots x_1 and x_2 is equal to 4 and satisfying the relation $\frac{x_1}{x_1-1}+\frac{x_2}{x_2-1}=2$ is

(a) $x^2 - 2x + 4 = 0$ (b) $x^2 - 4x + 4 = 0$
(c) $x^2 + 2x + 4 = 0$ (d) $x^2 + 4x + 4 = 0$

40. Number of values of x satisfying the pair of quadratic equations $x^2 - px + 20 = 0$ and $x^2 - 20x + p = 0$ for some $p \in R$, is

(a) 1 (b) 2 (c) 3 (d) 4

SOLUTIONS

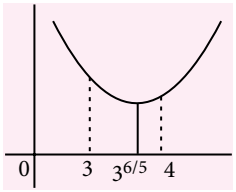
1. (d) : Let $\log_3 n = x$

$\therefore\ y = 5x^2 - 12x + 9$

y is minimum at $x = -\frac{b}{2a} = \frac{12}{10} = \frac{6}{5}$

Here $\log_3 n = \frac{6}{5} \Rightarrow n = 3^{6/5} \cong 3.70$

which is not natural. Hence minimum occurs at the closest integer. Now $4 > 3^{6/5}$



$\Rightarrow\ 4^5 > 3^6$

$\Rightarrow\ 1024 > 729$, which is true

2. (c) : If roots of equation $x^2 + px + q = 0$ are α and β, then $\alpha + \beta = -p$ and $\alpha\beta = q$

and if roots of equation $x^2 + px - r = 0$ are γ, δ then $\gamma + \delta = -p$, $\gamma\delta = -r$

Now, $(\alpha - \gamma)(\alpha - \delta) = \alpha^2 - (\gamma + \delta)\alpha + \gamma\delta = \alpha^2 + p\alpha - r$

$= -(q + r)$ $\{\because \alpha^2 + p\alpha + q = 0 \Rightarrow \alpha^2 + p\alpha = -q\}$.

3. (b) : $\alpha + \beta = -3$; $\alpha\beta = 1$.

$\alpha^2 + 3\alpha + 1 = 0$ and $\beta^2 + 3\beta + 1 = 0$, where

$\alpha^2 = -(3\alpha + 1)$ and $\beta^2 = -(3\beta + 1)$

Let $E=\frac{\alpha^2}{(1+\beta)^2}+\frac{\beta^2}{(\alpha+1)^2} = \frac{\alpha^2}{1+2\beta+\beta^2}+\frac{\beta^2}{1+2\alpha+\alpha^2}$

$$=\left(\frac{-(3\alpha+1)}{-\beta}\right)+\left(\frac{-(1+3\beta)}{-\alpha}\right)$$

$$=\frac{1+3\alpha}{\beta}+\frac{1+3\beta}{\alpha}=\frac{\alpha(1+3\alpha)+\beta(1+3\beta)}{\alpha\beta} \quad (\text{as } \alpha\beta=1)$$

$= 3(\alpha^2 + \beta^2) + (\alpha + \beta) = 3(9 - 2) + (-3)$

$= 21 - 3 = 18.$

4. (b) : If roots are real and distinct, then $\Delta > 0$

$\therefore$ For equation $x^2 + 6x + a = 0$.

$36 - 4a > 0$ or $a < 9$ ...(i)

Also, $\alpha - \beta \le 4 \Rightarrow (\alpha - \beta)^2 \le 16$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta \le 16$

$\Rightarrow 4a \ge 20 \Rightarrow a \ge 5.$

5. (d) : $\cot^4 x - 2(1 + \cot^2 x) + a^2 = 0$

$\Rightarrow \cot^4 x - 2\cot^2 x + a^2 - 2 = 0$

$\Rightarrow (\cot^2 x - 1)^2 = 3 - a^2$

To have atleast one solution $3 - a^2 \ge 0$

$\Rightarrow a^2 - 3 \le 0 \Rightarrow a \in [-\sqrt{3}, \sqrt{3}]$

Integral values of a are -1, 0, 1

$\therefore$ Required sum = 0.

6. (b) : $4x^2 - 4(5x + 1) + p^2 = 0$

$\Rightarrow 4x^2 - 20x + (p^2 - 4) = 0$

Now, two roots are α, $\alpha + 2$

$\therefore\ 2\alpha+2=\frac{20}{4}=5 \Rightarrow \alpha+1=\frac{5}{2} \Rightarrow \alpha=\frac{3}{2}$

and $\alpha(\alpha+2)=\frac{p^2-4}{4} \Rightarrow \frac{3}{2}\left(\frac{3}{2}+2\right)=\frac{p^2-4}{4}$

$\Rightarrow \frac{3}{2}\cdot\frac{7}{2}=\frac{p^2-4}{4} \Rightarrow 21=p^2-4$

$\Rightarrow p^2 = 25 \Rightarrow p = \pm 5$

7. (c) : Let the common root be α

$\therefore\ (1 - 2k)\alpha^2 - 6k\alpha - 1 = 0$

and $k\alpha^2 - \alpha + 1 = 0$

$$\Rightarrow \frac{\alpha^2}{-6k-1}=\frac{\alpha}{-k-(1-2k)}=\frac{1}{-(1-2k)+6k^2}$$

$$\Rightarrow \frac{\alpha^2}{-(6k+1)}=\frac{\alpha}{k-1}=\frac{1}{6k^2+2k-1}$$

$$\Rightarrow \alpha^2=\frac{-(6k+1)}{6k^2+2k-1},\ \alpha=\frac{k-1}{6k^2+2k-1}$$

$\Rightarrow (k - 1)^2 = -(6k + 1)(6k^2 + 2k - 1)$

$\Rightarrow -k^2 + 2k - 1 = 36k^3 + 12k^2 - 6k + 6k^2 + 2k - 1$

$\Rightarrow 36k^3 + 19k^2 - 6k = 0 \Rightarrow k(36k^2 + 19k - 6) = 0$

$\because\ k \ne 0\ \therefore\ 36k^2 + 19k - 6 = 0$

$\Rightarrow 36k^2 + 27k - 8k - 6 = 0$

$\Rightarrow 9k(4k + 3) - 2(4k + 3) = 0$

$\Rightarrow (4k + 3)(9k - 2) = 0$

$$\Rightarrow k=\frac{-3}{4},\ k=\frac{2}{9}$$

8. (b) : $|x - p| = x - p$ (Since $x \geq p$)

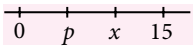
$|x - 15| = 15 - x$ (Since $x \leq 15$)

$|x - (p + 15)| = (p + 15) - x$ (Since $15 + p \geq x$)

$\therefore$ The given expression reduces to

$E = x - p + 15 - x + p + 15 - x$

$\Rightarrow E = 30 - x$



$\therefore$ E_{min} occurs when $x = 15$

$\therefore$ $E_{min} = 15$.

9. (d) : $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$,

then $xy + yz + zx = 5$

$\therefore$ $y + z = 4 - x$ ...(i)

and $yz = 5 - x(4 - x)$ ...(ii)

Now, if y and z are roots of any quadratic, then

$f(t) = t^2 - (4 - x)t + 5 - x(4 - x)$

If t is real then $D \geq 0$

$\Rightarrow (4 - x)^2 - 4[5 - x(4 - x)] \geq 0$

$\Rightarrow 16 + x^2 - 8x - 20 + 4x(4 - x) \geq 0$

$\Rightarrow 16 + x^2 - 8x - 20 + 16x - 4x^2 \geq 0$

$\Rightarrow -3x^2 + 8x - 4 \geq 0 \Rightarrow 3x^2 - 8x + 4 \leq 0$

$\Rightarrow x \in \left[\frac{2}{3}, 2\right]$

10. (c) : $a[x^2 - (b + c)x + bc] + b[x^2 - (c + a)x + ac] + c[x^2 - (a + b)x + ab] = 0$

$\Rightarrow (a + b + c)x^2 - 2x(ab + bc + ca) + 3abc = 0$

Now, $D = 4(ab + bc + ca)^2 - 12abc(a + b + c)$

$= 4[a^2b^2 + b^2c^2 + c^2a^2 + 2abc(a + b + c) - 3abc(a + b + c)]$

$= 4[a^2b^2 + b^2c^2 + c^2a^2 - abc(a + b + c)]$

$= 2[(ab - bc)^2 + (bc - ca)^2 + (ca - ab)^2] > 0$

11. (c) : If $\tan\theta$, $\cot\theta$ are roots of the equation $x^2 + 2x + 1 = 0$, then

$\tan\theta + \cot\theta = -2$, then $\tan\theta = -1$ and $\cot\theta = -1$

The least value of $x^2 + \tan\theta x + \cot\theta$

$$= x^2 - x - 1 = \left(x^2 - x + \frac{1}{4}\right) - \frac{5}{4}$$

$$= \left(x - \frac{1}{2}\right)^2 + \left(-\frac{5}{4}\right)$$

$\therefore$ Least value of function is $-\frac{5}{4}$

12. (d) : If α, β, γ are the roots of the equation, then $\alpha + \beta + \gamma = 2$. Also, $\alpha + \beta = 0$ (where α, β are additive inverse)

$\therefore$ $\gamma = 2$ which must satisfy the given equation

$\therefore$ $a = -5$

13. (a) : $a^2 + b^2 + c^2 = 1 - 2\Sigma ab$ ...(i)

Hence $a^2 + b^2 + c^2 \geq ab + bc + ca$

$\Rightarrow 1 - 2\Sigma ab \geq \Sigma ab$ [Using (i)]

$\Rightarrow 1 \geq 3\Sigma ab \therefore \Sigma ab \leq \frac{1}{3}$

14. (b) : We have, $\sqrt[3]{x + 3p + 1} = \sqrt[3]{x} + 1$, Let $\sqrt[3]{x} = h$

$\Rightarrow \sqrt[3]{x + 3p + 1} = h + 1$

$\Rightarrow x + 3p + 1 = h^3 + 3h^2 + 3h + 1$

$\Rightarrow h^3 + 3p + 1 = h^3 + 3h^2 + 3h + 1$

$\Rightarrow 3h^2 + 3h - 3p = 0$

For real solution $D \geq 0$

$\therefore$ $b^2 - 4ac = 1 + 4p \geq 0$

or $p \geq -1/4$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique solution.

1		5						7
	8	2					1	
		4		3		2	9	5
			2		9			
		8				5		
			4		3			
8	6	3		9		4		
	9					1	8	
4						9		6

2		9						4
	7	3					8	
		1		5		7	2	3
			1		3			
		6				2		
			4		5			
6	3	8		4		9		
	9					8	5	
1						3		7

						3		7
						6		4
			3		7			
				9				
			6		4			

1		9						4
	7	4				9		
		5		4		7	2	1
			1		8			
		3				2		
			9		2			
9	2	8		6		3		
	6					8	4	
5						1		9

2		9						7
	7	1					8	
		6		9		1	5	2
			1		4			
		3				5		
			8		9			
8	3	4		6		9		
	9					6	2	
1						8		3

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

Alternate solution :

$\sqrt[3]{x+3p+1}+(-\sqrt[3]{x})+(-1)=0$

If $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

$\Rightarrow x + 3p + 1 - x - 1 = 3\,[(x + 3p + 1)(x)]^{1/3}$

$\Rightarrow 3p = 3[(x + 3p + 1)(x)]^{1/3}$

$\Rightarrow p^3 = x(x + 3p + 1)$

$\therefore x^2 + (3p + 1)x - p^3 = 0$

For real roots $D \geq 0$

$\Rightarrow 4p^3 + 9p^2 + 6p + 1 \geq 0$

$\Rightarrow (p + 1)^2 (4p + 1) \geq 0 \Rightarrow p \geq -1/4$.

15. (c) : Given α, β, γ are roots of the equation $x^3 + qx + q = 0$

Now, $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1}$

$$= -\frac{1}{\gamma} - \frac{1}{\alpha} - \frac{1}{\beta} = -\left(\frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}\right) = 1$$

$\{\because \alpha\beta + \beta\gamma + \gamma\alpha = q, \alpha\beta\gamma = -q\}$

16. (b) : Let $QR = x$

Then the diameters are $2, x + 2, x + 3$

$$\Rightarrow \frac{2^2 + (x+3)^2}{2} = (x+2)^2$$

$\Rightarrow 2(x + 2)^2 = 2^2 + (x + 3)^2$

$\Rightarrow 2(x^2 + 4 + 4x) = 4 + (x^2 + 6x + 9)$

$\Rightarrow x^2 + 2x - 5 = 0 \Rightarrow x = \sqrt{6} - 1$

17. (c) : From 1st equation, $\cos x = \frac{1 \pm \sqrt{13}}{6}$

$$\text{Now, } \cos 2x = 2\cos^2 x - 1 = \frac{2}{36}(1 + 13 \pm 2\sqrt{13}) - 1$$

$$= \frac{14 \pm 2\sqrt{13} - 18}{18} = \frac{\pm 2\sqrt{13} - 4}{18} = \frac{\pm\sqrt{13} - 2}{9}$$

$$\therefore \cos 2x_1 = \frac{\sqrt{13} - 2}{9} \text{ and } \cos 2x_2 = -\frac{(\sqrt{13} + 2)}{9}$$

Now from 2nd equation,

$a\cos^2 2x + b\cos 2x - 1 = 0$

$$\therefore \cos 2x_1 \cdot \cos 2x_2 = -\frac{1}{a}$$

$$\therefore -\frac{1}{a} = -\left(\frac{13 - 4}{81}\right) = -\frac{1}{9} \Rightarrow a = 9$$

$$\text{and } \cos 2x_1 + \cos 2x_2 = -\frac{b}{a} = -\frac{b}{9}$$

$$\therefore -\frac{b}{9} = -\frac{4}{9} \Rightarrow b = 4$$

$\therefore a + b = 13.$

18. (c) : Let $2^x = t$, then

$t^2 + (k - 3)t + (k - 4) = 0$

$$\Rightarrow t = \frac{-(k-3) \pm \sqrt{(k-3)^2 - 4(k-4)}}{2}$$

$$\Rightarrow t = \frac{-(k-3) \pm (k-5)}{2} \Rightarrow t = \frac{-2k+8}{2} \Rightarrow t = -k + 4$$

$\therefore x$ is non positive, then $x \leq 0$

$\therefore 0 < 2^x \leq 1$

$\Rightarrow 0 < -k + 4 \leq 1 \Rightarrow 3 \leq k < 4$

$\therefore$ Largest integral value of k is 3.

19. (c) : $x^3 - 2x^2 + 4x + 5074 = (x - r_1)(x - r_2)(x - r_3)$

Put $x = -2$

$\therefore -8 - 8 - 8 + 5074 = -(2 + r_1)(2 + r_2)(2 + r_3)$

$\therefore 5050 = -(2 + r_1)(2 + r_2)(2 + r_3)$

or $(2 + r_1)(2 + r_2)(2 + r_3) = -5050.$

20. (d) : $\log_2\alpha + \log_2\beta = -4$; $\log_2\alpha \cdot \log_2\beta = -1$

$$\text{Now } \log_\beta \alpha + \log_\alpha \beta = \frac{\log_2 \alpha}{\log_2 \beta} + \frac{\log_2 \beta}{\log_2 \alpha}$$

$$= \frac{(\log_2 \alpha)^2 + (\log_2 \beta)^2}{\log_2 \alpha \cdot \log_2 \beta}$$

$= -[(\log_2\alpha + \log_2\beta)^2 - 2\log_2\alpha \cdot \log_2\beta]$

$= -[16 + 2] = -18$

21. (d) : $f(x) = (2 + b + b^2)x^2 - 2\sqrt{2}(2b + 1)x + 8$

$$\text{Min value of } f(x) = \frac{-D}{4a} = \frac{-[8(2b+1)^2 - 32(2 + b + b^2)]}{4(b^2 + b + 2)}$$

$$\Rightarrow m(b) = \frac{56}{4(b^2 + b + 2)}$$

Maximum value of $m(b)$ is obtained, when minimum value of $b^2 + b + 2$ is obtained.

Minimum value of $b^2 + b + 2$

$$= \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) + 2 = \frac{1}{4} - \frac{1}{2} + 2 = \frac{7}{4}$$

$$\therefore \text{Maximum value of } m(b) = \frac{56}{4 \times \frac{7}{4}} = 8$$

22. (d)

23. (b) : $x^2 - 1088x + 295680 = 0$

Let α and β be the the roots of given equation.

Also let $\alpha = 16k_1$ and $\beta = 16k_2$

(As H.C.F. of roots is 16)

Now, $\alpha\beta$ = (H.C.F (α, β)) (L.C.M (α, β))

$\Rightarrow$ 295680 = 16(L.C.M. (α, β))

$\Rightarrow$ $\text{L.C.M}(\alpha,\beta)=\dfrac{295680}{16}=18480$

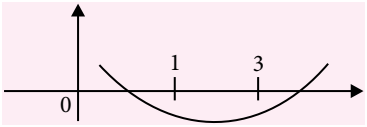
24. (d) : Using R.M.S. $\geq$ AM in a, b, c, we get

$$\sqrt{\frac{a^2+b^2+c^2}{3}}\geq\frac{a+b+c}{3}$$

$$\Rightarrow \sqrt{3}\geq a+b+c \Rightarrow a+b+c\leq\sqrt{3}$$

25. (b) : The given equation is $x^2 - (4a + 3)x + 3a(a + 3)$

Now, $f(1) < 0$ and $f(3) < 0$



$\Rightarrow$ $(1 - 3a)(1 - a - 3) < 0$

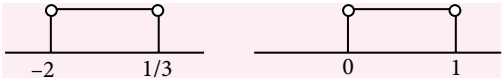
$\Rightarrow$ $1 - a - 3 - 3a + 3a^2 + 9a < 0$

$\Rightarrow$ $3a^2 + 5a - 2 < 0 \Rightarrow 3a^2 + 6a - a - 2 < 0$

$\Rightarrow$ $3a(a + 2) - (a + 2) < 0$

$\Rightarrow$ $(a + 2)(3a - 1) < 0$

Again, $(3 - 3a)(-a) < 0 \Rightarrow (a - 1)a < 0$



$\Rightarrow$ $0 < a < 1$

Hence, $a\in\left(0,\dfrac{1}{3}\right)$

26. (a) : Let n, $n + 1$, $n + 2$ are the roots of the given equation.

$\therefore$ Sum $= 3(n + 1) = -a$

$\Rightarrow$ $a^2 = 9(n + 1)^2$ ...(i)

Let sum of the roots taken 2 at a time $= b$

$\therefore$ $n(n + 1) + (n + 1)(n + 2) + (n + 2)(n) + 1 \quad b + 1$

(adding 1 on both sides)

$\Rightarrow$ $n^2 + n + n^2 + 3n + 2 + n^2 + 2n + 1 = b + 1$

$\Rightarrow$ $b + 1 = 3n^2 + 6n + 3 = 3(n + 1)^2$

$\Rightarrow$ $b + 1 = 3(n + 1)^2 = \dfrac{a^2}{3}$ [Using (i)]

$\therefore$ $\dfrac{a^2}{b+1}=3$

27. (c) : Given, $x^3 + 4x - 8 = 0$

Let $y = x^7 + 64x^2$

$= \underbrace{x^4(x^3+4x-8)}_{\text{zero}} - 4x^5 + 8x^4 + 64x^2$

$= -4x^5 + 8x^4 + 64x^2$

$= -4x^2\underbrace{(x^3+4x-8)}_{\text{zero}} + 8x^4 + 16x^3 + 32x^2$

$= 8x^4 + 16x^3 + 32x^2$

$= 8x\underbrace{(x^3+4x-8)}_{\text{zero}} + 16x^3 + 64x$

$= \underbrace{16(x^3+4x-8)}_{\text{zero}} + 128 = 128$

28. (b) : $\sum\tan\alpha = p$, $\sum\tan\alpha\cdot\tan\beta = 0$, $\prod\tan\alpha = r$

Now, $\sec^2\alpha\cdot\sec^2\beta\cdot\sec^2\gamma$

$= (1 + \tan^2\alpha)(1 + \tan^2\beta)(1 + \tan^2\gamma)$

$=1+\sum(\tan^2\alpha)+\sum(\tan^2\alpha\cdot\tan^2\beta)+\tan^2\alpha\cdot\tan^2\beta\cdot\tan^2\gamma$

Now, $\sum\tan^2\alpha=\left(\sum\tan\alpha\right)^2-2\sum\tan\alpha\cdot\tan\beta=p^2$

$\sum\tan^2\alpha\cdot\tan^2\beta=\left(\sum\tan\alpha\cdot\tan\beta\right)^2$

$-2\tan\alpha\cdot\tan\beta\cdot\tan\gamma\left(\sum\tan\alpha\right)$

$= 0 - 2rp$

Also, $\prod\tan^2\alpha=r^2$

$\therefore$ $\prod\sec^2\alpha = 1 + p^2 - 2rp + r^2 = 1 + (p - r)^2$

29. (c) : $\alpha\beta = \alpha^2\beta^2$ ...(1)

and $\alpha^2 + \beta^2 = \alpha + \beta$ ...(2)

Hence $\alpha\beta(1 - \alpha\beta) = 0 \Rightarrow \alpha = 0$ or $\beta = 0$ or $\alpha\beta = 1$

If $\alpha = 0$, then from (2), $\beta = 0$ or $\beta = 1$

$\Rightarrow$ Roots are (0, 0) or (0, 1)

If $\beta = 0$, then $\alpha = 0$ or $\alpha = 1$

$\Rightarrow$ Roots are (0, 0) or (1, 0)

If $\beta = \dfrac{1}{\alpha}$, then $\alpha^2+\dfrac{1}{\alpha^2}=\alpha+\dfrac{1}{\alpha}$

$\Rightarrow$ $\left[\alpha+\dfrac{1}{\alpha}\right]^2-2=\alpha+\dfrac{1}{\alpha}$

Hence, $t^2 - t - 2 = 0$

$\Rightarrow$ $(t - 2)(t + 1) = 0 \Rightarrow t = 2$ or $t = -1$

If $t = 2 \Rightarrow \alpha = 1$ and $\beta = 1$, if $t = -1$, then roots are imaginary (ω or ω^2).

30. (b) : Let α, β are the roots of the given equation.

$$\therefore\ \alpha,\beta=\frac{2b(a+c)\pm\sqrt{4b^2(a+c)^2-4(a^2+b^2)(b^2+c^2)}}{2(a^2+b^2)}$$

$$=\frac{b(a+c)\pm\sqrt{b^2(a^2+2ac+c^2)-(a^2b^2+a^2c^2+b^4+b^2c^2)}}{a^2+b^2}$$

$$= \frac{b(a+c) \pm \sqrt{-(b^4 - 2b^2ac + a^2c^2)}}{a^2 + b^2}$$

$$= \frac{b(a+c) \pm \sqrt{-(b^2 - ac)^2}}{a^2 + b^2}$$

In order that roots may be real $D \geq 0 \Rightarrow D = 0$

$\Rightarrow b^2 - ac = 0 \Rightarrow b^2 = ac$

Hence roots are co-incident and equal to

$\frac{b(a+c)}{a^2 + ac} = \frac{b}{a}$ which satisfies B.

31. (c) : We have, $x^2 - bx + c = 0$

Let the roots are α and $\alpha + 1$

$\therefore$ Sum of roots $= 2\alpha + 1 = b$ ...(i)

Product $= \alpha(\alpha + 1) = c$ ...(ii)

From (i), $\alpha = (b - 1)/2$

Put the value of α in (ii), we get

$$\Rightarrow \left(\frac{b-1}{2}\right)^2 + \left(\frac{b-1}{2}\right) = c$$

$\Rightarrow b^2 - 2b + 1 + 2b - 2 = 4c \Rightarrow b^2 - 4c = 1$

32. (d) : $\sin^2 x = \frac{(k+2) \pm \sqrt{(k+2)^2 + 4(k+3)}}{2}$

$$= \frac{(k+2) \pm \sqrt{k^2 + 8k + 16}}{2} = \frac{(k+2) \pm (k+4)}{2}$$

$\Rightarrow \sin^2 x = k + 3$ or -1 (rejected)

$\Rightarrow 0 \leq k + 3 \leq 1 \Rightarrow -3 \leq k \leq -2$

33. (c) : $|x| - |x - 1| + |x - 2| = \sqrt{5}$

For $x \geq 2$, $x - (x - 1) + x - 2 = \sqrt{5}$

$\Rightarrow x = 1 + \sqrt{5}$

For $1 \leq x < 2$, $x - (x - 1) + (2 - x) = \sqrt{5}$

$\Rightarrow 3 - x = \sqrt{5}$

$\Rightarrow x = 3 - \sqrt{5}$ (No solution)

For $0 \leq x < 1$, $x - (1 - x) + 2 - x = \sqrt{5}$

$\Rightarrow x = \sqrt{5} - 1$ (No solution)

For $x < 0$, $-x - (1 - x) + 2 - x = \sqrt{5}$

$\Rightarrow x = 1 - \sqrt{5}$

Hence, $x = \sqrt{5} + 1$ or $x = 1 - \sqrt{5}$

34. (a) : Given, $a + b + c = 0$

and $a^2 + b^2 + c^2 = 2$

$\Rightarrow (a^2 + b^2 + c^2)^2 = 4$

$\Rightarrow a^4 + b^4 + c^4 + 2[a^2b^2 + b^2c^2 + c^2a^2] = 4$

Let $a^4 + b^4 + c^4 = E$

Hence, $E + 2[(ab + bc + ca)^2 - 2abc(a + b + c)] = 4$

$\therefore E + 2(ab + bc + ca)^2 = 4$ ...(i) (As $a + b + c = 0$)

Again, $(a + b + c)^2 = 0 \Rightarrow \sum a^2 + 2\sum ab = 0$

$\Rightarrow 2 + 2(ab + bc + ca) = 0$

$\Rightarrow ab + bc + ca = -1$

$\therefore$ From (i), $E + 2 = 4$

$\therefore E = 2$

35. (d) : $4^x + 4.6^x = 5.9^x$

$$\Rightarrow \left(\frac{4}{6}\right)^x + 4 = 5\left(\frac{9}{6}\right)^x \Rightarrow \left(\frac{2}{3}\right)^x + 4 = 5 \cdot \left(\frac{3}{2}\right)^x$$

Let $\left(\frac{2}{3}\right)^x = t$, then $t + 4 = 5\left(\frac{1}{t}\right) \Rightarrow t^2 + 4t - 5 = 0$

$\Rightarrow (t + 5)(t - 1) = 0$

$\Rightarrow t \neq -5$ (As t can't be negative)

$\therefore t = 1 \Rightarrow \left(\frac{2}{3}\right)^x = 1 \Rightarrow x = 0$ is the solution.

36. (a) : $a\alpha^2 + b\alpha + c = a\alpha^2 + p\alpha + q$ [$\because f(\alpha) = g(\alpha)$]

$\therefore \alpha = \frac{q - c}{b - p}$ ...(i)

A.M. of roots of $f(x) = 0$ and $g(x) = 0$ is

$\frac{-b/a - p/a}{4} = -\frac{b + p}{4a}$ (Number of roots is 4)

Since it is given that discriminants are equal

$\therefore b^2 - 4ac = p^2 - 4aq$

$\Rightarrow b^2 - p^2 = 4ac - 4aq$

$\Rightarrow \frac{b + p}{4a} = \frac{c - q}{b - p} = -\alpha$ [From (i)]

$\therefore \alpha$ = A.M. of roots of $f(x) = 0$ and $g(x) = 0$

37. (b) : We need $x^2 + 2bx + 1 = 2ax + 2ab$ not to have any real solutions, implying that the discriminant is less than or equal to zero. Actually calculating the discriminant and simplifying, we get $a^2 + b^2 < 1$, which describes a circle of area π in the ab plane]

38. (a) : The y-intercept is at $x = 0$, so we have $c = 2$, meaning that the product of the roots is -2. We know that a is the sum of the roots. The average of the roots is equal to the product, so the sum of the roots is -6, and $a = 6$. Finally, $1 + a + b + c = -2$ as well, so we have $1 + 6 + b + 2 = -2 \Rightarrow b = -11$

39. (a) : Given, $x_1 x_2 = 4 \Rightarrow x_2 = \frac{4}{x_1}$

Consider $\frac{x_1}{x_1 - 1} + \frac{x_2}{x_2 - 1} = 2$

$$\Rightarrow \frac{x_1}{x_1 - 1} + \frac{\frac{4}{x_1}}{\frac{4}{x_1} - 1} = 2 \Rightarrow \frac{x_1}{x_1 - 1} + \frac{4}{4 - x_1} = 2$$

$\Rightarrow 4x_1 - x_1^2 + 4x_1 - 4 = 2(x_1 - 1)(4 - x_1)$

$\Rightarrow x_1^2 - 2x_1 + 4 = 0 \Rightarrow x^2 - 2x + 4 = 0.$

40. (c) : $x^2 - px + 20 = 0$

and $x^2 - 20x + p = 0$

If $p = 20$, then both the quadratic equations are identical.

Hence, $x = 10 + 4\sqrt{5}$

or $x = 10 - 4\sqrt{5}$ satisfy both.

If $p \neq 20$, then $x^2 - px + 20 = x^2 - 20x + p$

$\Rightarrow (20 - p)x + (20 - p) = 0$

$\Rightarrow x = -1$ and $p = -21$

Hence, there are 3 values of x

i.e., $\left\{10 + 4\sqrt{5},\ 10 - 4\sqrt{5}, -1\right\}$

Abel Prize Winners

The **Abel Prize** is a Norwegian prize awarded annually by the Government of Norway to one or more outstanding Mathematicians.

The Abel Prize recognizes contributions to the field of Mathematics that are of extraordinary depth and influence.

The Norwegian Academy of Science and Letters awards the Abel Prize based on a recommendation from the Abel committee. The prize carries a cash award of 6 million NOK and has been awarded annually since 2003.

In addition to honouring outstanding Mathematicians, the Abel Prize shall contribute towards raising the status of Mathematics in society and stimulating the interest of children and young people in Mathematics.

Robert Langlands
Canada / United States
2018

"For his visionary program connecting representation theory to number theory."

Yves Meyer
France
2017

"For his pivotal role in the development of the Mathematical theory of wavelets."

Andrew Wiles
United Kingdom
2016

"For his stunning proof of Fermat's Last Theorem by way of the modularity conjecture for semistable elliptic curves, opening a new era in number theory."

MOCK TEST PAPER 2019

JEE MAIN

Time : 1 hr 15 min. **Series - 1**

The entire syllabus of Mathematics of JEE MAIN is being divided in to eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

	Topic	Syllabus In Detail
UNIT NO. 1	Sets, Relations & Functions	Sets and their representation; Union, intersection and compliment of sets and their algebraic properties; Power set. Ordered pairs, Cartesian product of sets. Definition of relation. Domain and range of a relation. Types of relations, equivalence relations. Function as a special kind of relation from one set to another. Domain, co-domain and range of a function. One-one, into and onto functions, Composition of functions. Real valued function of a real variable, domain and range of these functions, constant, identity, polynomial, rational, modulus, Signum and greatest integer functions with their graphs, sum, difference, product and quotients of functions.
	Trigonometry	Measurement of Trigonometric Angles. Definition of Trigonometric functions, Associated angles
	Co-ordinate Geometry-2D	Cartesian coordinates, distance between two points, section formulae, shift of origin. Locus problems.

1. If $(A \cap B) = \phi$, then $A \cup B'$ equals

(a) A (b) B (c) A' (d) B'

2. In a class of 100 students, 55 have passed in Mathematics and 67 in physics. If each of the students pass at least in one of the subjects, then the number of students who have passed in physics only is

(a) 22 (b) 33 (c) 10 (d) 45

3. Let $Y = \{1, 2, 3, 4, 5\}$, $A = \{1, 2\}$, $B = \{3, 4, 5\}$, then $(Y \times A) \cap (Y \times B)$ is

(a) Y (b) A (c) B (d) ϕ

4. If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is

(a) 2^9 (b) 9^2 (c) 3^3 (d) $2^9 - 1$

5. Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$, then the relation is

(a) reflexive and transitive only

(b) reflexive only

(c) an equivalence relation

(d) reflexive and symmetric only

6. A relation ρ on the set of real numbers R is defined as follows: " $x\rho y$ if and only if $xy > 0$. Then which of the following is true?

(a) ρ is reflexive and symmetric

(b) ρ is symmetric and transitive

(c) ρ is transitive only

(d) ρ is an equivalence relation.

7. The range of the function

$$y = f(x) = \log_2 \frac{\sin x - \cos x + 3\sqrt{2}}{2} \text{ is}$$

(a) $0 < y \leq 3$ (b) $0 < y < 3$

(c) $1 \leq y \leq 2$ (d) $1 < y < 2$

8. The domain of the function

$f(x) = \log_4 (\log_5(\log_3(18x - x^2 - 17)))$ is

(a) $x \in (4, 5)$ (b) $x \in (0, 10)$

(c) $x \in (8, 10)$ (d) None of these

By : Sankar Ghosh, S.G.M.C, Mob. 09831244397.

9. The value of 'a' for which $f:R\to R, f(x)=\dfrac{ax+7}{x^2+4}$ is invertible is

(a) $(0, 2)$ (b) $(1, \infty)$

(c) $(0, \infty)$ (d) None of these

10. Let $f:[-3,3]\to R$, where $f(x)=x^3+\sin x+\left[\dfrac{x^2+2}{a}\right]$ be an odd function, then values of a is (where $[\cdot]$ represents greatest integer function)

(a) less than 11 (b) 11

(c) greater than 11 (d) None of these

11. Let $f: R \to R$ where $f(x) = x^2 + 5x + 9$, then $f^{-1}(8) =$

(a) $\left\{\dfrac{-5+\sqrt{20}}{2}, \dfrac{-5-\sqrt{21}}{2}\right\}$

(b) $\left\{\dfrac{-5+\sqrt{21}}{2}, \dfrac{-5-\sqrt{21}}{2}\right\}$

(c) $\left\{\dfrac{5-\sqrt{21}}{2}, \dfrac{21-\sqrt{5}}{2}\right\}$

(d) None of these

12. $f:R\to\left\{\dfrac{2}{3}\right\}\Rightarrow$ R, $f(x)=\dfrac{3x+5}{2x-3}$. Let $f_2(x) = f(f(x))$, $f_3(x) = f(f_2(x))$, ..., $f_n(x) = f(f_{n-1}(x))$, then $f_{2018}(x) + f_{2019}(x) =$

(a) $\dfrac{2x^2+5}{2x-3}$ (b) $\dfrac{x^2+5}{2x-3}$

(c) $\dfrac{2x^2-5}{2x-3}$ (d) $\dfrac{x^2-5}{2x-3}$

13. If $f: [1, \infty) \to [2, \infty)$, $f(x)=x+\dfrac{1}{x}$, then $f^{-1}(x)=$

(a) $\dfrac{x+\sqrt{x^2-4}}{2}$ (b) $\dfrac{x}{1+x^2}$

(c) $\dfrac{x-\sqrt{x^2-4}}{2}$ (d) $1+\sqrt{x^2-4}$

14. Let $f(x) = \log|x|$ and $g(x) = \sin x$. If A is the range of $f(g(x))$ and B is the range of $g(f(x))$, then

(a) $A \cup B = (-\infty, 1]$ (b) $A \cup B = (-\infty, \infty)$

(c) $A \cap B = [-1, 1]$ (d) $A \cap B = [0, 1]$

15. The domain of $f(x)=\sqrt{\dfrac{4-x^2}{2+[x]}}$ is

(a) $[-2, 2]$ (b) $(-1, 2)$

(c) $(-\infty, -2) \cup [-1, 2]$ (d) $(-\infty, -2) \cup (-1, 2)$.

16. Let $f: (-1, 1) \to B$, $f(x)=\tan^{-1}\dfrac{2x}{1-x^2}$ is one-one and onto, then $B =$

(a) $\left[0,\dfrac{\pi}{2}\right]$ (b) $\left(0,\dfrac{\pi}{2}\right)$

(c) $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$ (d) $\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$

17. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain

(a) $\left[0,\dfrac{\pi}{2}\right]$ (b) $\left[-\dfrac{\pi}{4},\dfrac{\pi}{4}\right]$

(c) $\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$ (d) $[0, \pi]$

18. The function $f(x)=\sqrt{3}\sin 2x-\cos 2x+4$ is one-one in the interval

(a) $\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$ (b) $\left[-\dfrac{\pi}{4},\dfrac{\pi}{4}\right]$

(c) $\left[-\dfrac{\pi}{6},\dfrac{\pi}{3}\right]$ (d) $[-\pi, \pi]$

19. The range of $f(x)=\dfrac{x-[x]}{1-[x]+x}$ is given by

(a) $\left[0,-\dfrac{1}{2}\right]$ (b) $\left[-\dfrac{1}{2},0\right]$

(c) $\left[-\dfrac{1}{2},\dfrac{1}{2}\right]$ (d) $\left[0,\dfrac{1}{2}\right]$

20. The period of the function $f(\theta)=\sin\dfrac{\theta}{3}+\cos\dfrac{\theta}{2}$ is

(a) 3π (b) 6π (c) 9π (d) 12π

21. If $[\sin x]+[\sqrt{2}\cos x]=-3, x\in[0,2\pi]$ (where $[\cdot]$ denotes the greatest integer function), then x belongs to

(a) $\left[\dfrac{5\pi}{4},2\pi\right]$ (b) $\left(\dfrac{5\pi}{4},2\pi\right)$

(c) $\left(\pi,\dfrac{5\pi}{4}\right)$ (d) $\left[\pi,\dfrac{5\pi}{4}\right]$

22. The smallest possible values of x and y which satisfy $\tan(x - y) = 1$ and $\sec(x+y)=\dfrac{2}{\sqrt{3}}$ are

(a) $x=\dfrac{37\pi}{24}, y=\dfrac{7\pi}{24}$ (b) $x=\dfrac{15\pi}{24}, y=\dfrac{19\pi}{24}$

(c) $x=\dfrac{\pi}{3}, y=\dfrac{7\pi}{12}$ (d) $x=\dfrac{\pi}{4}, y=\dfrac{\pi}{2}$

23. If $e^{(\sin^2 x + \sin^4 x + \sin^6 x + \ldots \text{to } \infty)\log_e 2}$ satisfies the equation $x^2 - 9x + 8 = 0$, then the value of $\frac{\cos x}{\cos x + \sin x}, 0 < x < \frac{\pi}{2}$ is

(a) $\sqrt{3}+1$ (b) $\frac{1}{2}(\sqrt{3}+1)$

(c) $\frac{1}{2}(\sqrt{3}-1)$ (d) $\sqrt{3}-1$

24. If $\theta = \frac{\pi}{4n}$, where n is positive integer, then the value of $\tan\theta \tan 2\theta \tan 3\theta \ldots \tan(2n-1)\theta$ will be

(a) 1 (b) $\tan\theta$ (c) $\cot\theta$ (d) 0

25. In the interval $\left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$, the value of $\sqrt{\frac{1}{\cos^2\alpha} - \frac{2}{\cot\alpha}} =$

(a) $1 - \tan\alpha$ (b) $\tan\alpha - 1$

(c) $\cot\alpha - 1$ (d) $1 - \cot\alpha$

26. If the coordinate axes are shifted to the point $(-1, 2)$ without rotation, then the curve whose equation is $2x^2 + y^2 - 4x + 4y = 0$ will have the equation

(a) $2x^2 + y^2 - 8x + 8y + 18 = 0$

(b) $2x^2 + y^2 - 6 = 0$

(c) $2x^2 + y^2 + 8y + 10 = 0$

(d) $x^2 + 2y^2 + 8x - 8y + 18 = 0$

27. P, Q, R, S are the mid-points of the sides of a quadrilateral $ABCD$ in that order. If the co-ordinates of P, Q, R, S are $(0, 1)$, $(3, 5)$, $(4, 6)$ and (a, b) respectively, then

(a) $a = 1, b = 3$ (b) $a = 2, b = 3$

(c) $a = 1, b = 2$ (d) $a = 2, b = 4$

28. If all the three vertices of an isosceles right angle triangle be integral points and length of base is also an integer, then which of the point is never a rational point. (A point $p(x, y)$ is integer point if both x and y are integers and is rational if both x and y are rational).

(a) centroid (b) incentre

(c) circumcentre (d) orthocentre

29. The co-ordinate of the third vertex of an equilateral triangle whose two vertices are $(3, 4)$, $(-2, 3)$ are

(a) $(1, 7)$ (b) $(5, 1)$

(c) $\left(\frac{1+\sqrt{3}}{2}, \frac{7-5\sqrt{3}}{2}\right)$ or $\left(\frac{1-\sqrt{3}}{2}, \frac{7+5\sqrt{3}}{2}\right)$

(d) None of these

30. If the vertices of a triangle are $\left(4, \frac{1}{4}\right), \left(3, \frac{1}{3}\right), (1,1)$, then orthocentre of the triangle is

(a) $\left(\frac{1}{12}, 12\right)$ (b) $\left(12, \frac{1}{12}\right)$

(c) $\left(-\frac{1}{12}, -12\right)$ (d) None of these

SOLUTIONS

1. (d) : Here, $A \cup B' = (A \cup B') \cap U$

$= (A \cup B') \cap (A \cap B)'$

$[\because A \cap B = \phi \ \therefore \ (A \cap B)' = \phi' = U]$

$= (A \cup B') \cap (A' \cup B') = (A \cap A') \cup B'$

$= \phi \cup B' = B'$

2. (d) : Let A be the set of students who passed in Mathematics and B be the set of students who passed in physics.

$\therefore \ n(A \cup B) = 100$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\because \ n(A) = 55$ and $n(B) = 67$

$\Rightarrow \ 100 = 55 + 67 - n(A \cap B)$

$\Rightarrow \ n(A \cap B) = 122 - 100 = 22$

$\therefore$ The number of students who have passed in physics only $= n(B) - n(A \cap B) = 67 - 22 = 45$

3. (d) : $A = \{1, 2\}$, $B = \{3, 4, 5\}$, $A \cap B = \phi$

$(Y \times A) \cap (Y \times B) = Y \times (A \cap B) = Y \times \phi = \phi$

4. (a) : Here, $n(A) = 3$ and $n(B) = 3$

$\therefore$ Number of relations $= 2^{3 \times 3} = 2^9$

5. (a) : Given that, $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$

Reflexive property :

Since $(3, 3), (6, 6), (9, 9), (12, 12) \in R$ *i.e.*, $(a, a) \in R \ \forall \ a \in A$ is satisfied. Hence, R is reflexive.

Symmetric property :

Here, $(3, 6) \in R$ but $(6, 3) \notin R$

Hence, R is not symmetric.

Transitive property :

Here, we find that $(3, 6)$ and $(6, 12) \in R \ \Rightarrow \ (3, 12) \in R$ *i.e.* $(a, b), (b, c) \in R \ \Rightarrow \ (a, c) \in R \ \forall \ a, b, c \in A$ is satisfied. Hence, R is transitive.

6. (b) : Reflexive property :

$0 \in R$ (set of real numbers)

Then $(0, 0) \in \rho \ \Rightarrow \ 0 \cdot 0 > 0$ which is not true.

$\therefore \ \rho$ is not reflexive.

Symmetric property :

Let $x, y \in R$ such that $(x, y) \in \rho$.

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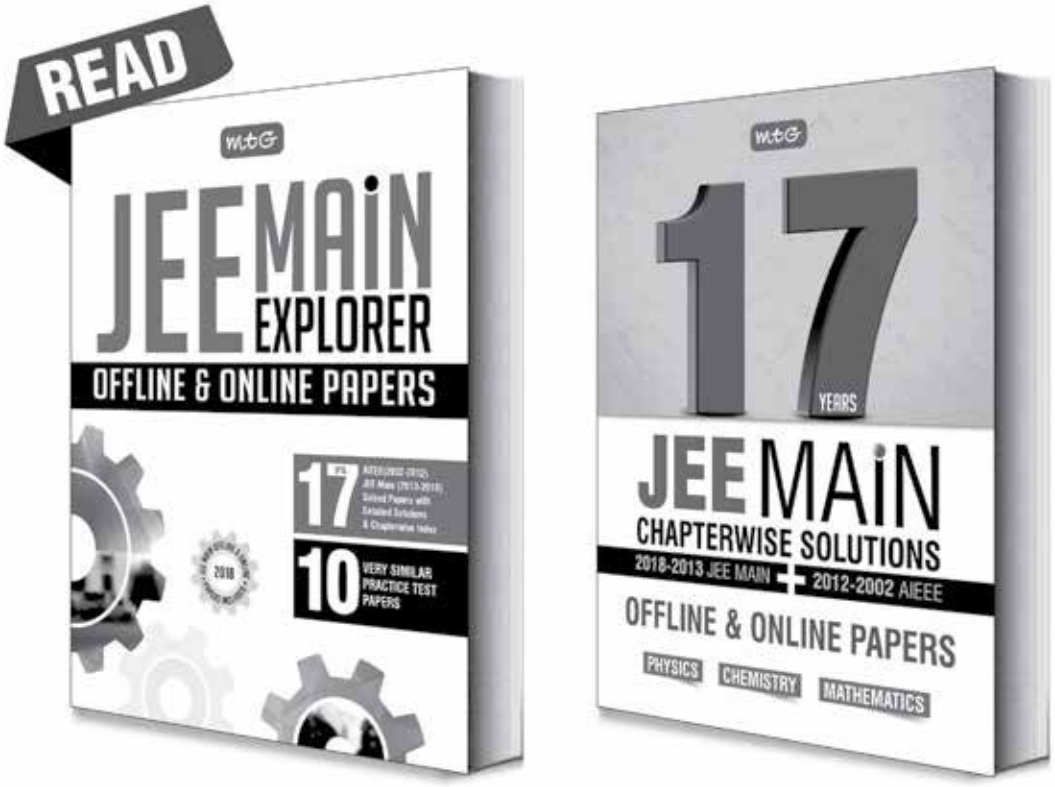

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$\Rightarrow xy > 0 \Rightarrow yx > 0 \Rightarrow (y, x) \in \rho$
Thus, $(x, y) \in \rho \Rightarrow (y, x) \in \rho$
Hence ρ is symmetric.
Transitive property :
Let $x, y, z \in R$ such that $(x, y), (y, z) \in \rho$.
$\Rightarrow xy > 0$ and $yz > 0$
$\Rightarrow (xy)(yz) > 0 \Rightarrow xzy^2 > 0$
$\Rightarrow xz > 0 \Rightarrow (x, z) \in \rho$
Thus, $(x, y), (y, z) \in \rho \Rightarrow (x, z) \in \rho$
Hence, ρ is transitive.

7. (c) : We have, $y = \log_2\left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$

$\Rightarrow \frac{1}{\sqrt{2}}(\sin x - \cos x) + 3 = 2^y$

$\Rightarrow \sin\left(x - \frac{\pi}{4}\right) + 3 = 2^y \Rightarrow \sin\left(x - \frac{\pi}{4}\right) = 2^y - 3$

We know, $-1 \le \sin\left(x - \frac{\pi}{4}\right) \le 1$

$\Rightarrow -1 \le 2^y - 3 \le 1 \Rightarrow 2 \le 2^y \le 4$
$\Rightarrow 2^1 \le 2^y \le 2^2 \Rightarrow 1 \le y \le 2$

8. (c) : We know $\log x$ is derived for all $x > 0$.
$\therefore f(x) = \log_4(\log_5(\log_3(18x - x^2 - 77)))$
$\Rightarrow \log_5(\log_3(18x - x^2 - 77)) > 0$
$\Rightarrow \log_3(18x - x^2 - 77) > 5^0$
$\Rightarrow 18x - x^2 - 77 > 3^1 \Rightarrow x^2 - 18x + 80 < 0$
$\Rightarrow (x - 8)(x - 10) < 0 \Rightarrow x \in (8, 10)$

9. (d) : Here, $f(x) = \frac{ax + 7}{x^2 + 4}$

$\therefore f'(x) = \frac{(x^2 + 4)a - (ax + 7)(2x)}{(x^2 + 4)^2} = \frac{-ax^2 - 14x + 4a}{(x^2 + 4)^2}$

Now, $f'(x) > 0$ for invertible function
$\therefore D = 196 - 4(4a)(-a) = 196 + 16a^2 > 0$

10. (c) : Given: $f(x) = x^3 + \sin x + \left[\frac{x^2 + 2}{a}\right]$ ... (i)

$\therefore f(-x) = -x^3 - \sin x + \left[\frac{x^2 + 2}{a}\right]$ ... (ii)

Now, $f(x) = -f(-x)$ [$\because f(x)$ is an odd function]

$\Rightarrow f(x) + f(-x) = 0 \Rightarrow 2\left[\frac{x^2 + 2}{a}\right] = 0$

$\Rightarrow \left[\frac{x^2 + 2}{a}\right] = 0 \Rightarrow 0 \le \frac{x^2 + 2}{a} < 1 \ \forall x \in [-3, 3]$

$\therefore a > 11$ [$\because$ Maximum of $x^2 + 2$ in $[-3, 3]$ is 11]

11. (b) : Given that $f : R \to R$ such that $f(x) = x^2 + 5x + 9$
Now, $f^{-1}(8) = \{x : f(x) = 8\}$
Now, $f(x) = 8$
$\Rightarrow x^2 + 5x + 9 = 8 \Rightarrow x^2 + 5x + 1 = 0$

$\Rightarrow x = \frac{-5 + \sqrt{21}}{2}, \frac{-5 - \sqrt{21}}{2} \in R$

Hence, $f^{-1}(8) = \left\{\frac{-5 + \sqrt{21}}{2}, \frac{-5 - \sqrt{21}}{2}\right\}$

12. (a) : Here $f(x) = \frac{3x + 5}{2x - 3}$

Now, $f_2(x) = f(f(x)) = \frac{3f(x) + 5}{2f(x) - 3} = x$
$\Rightarrow f_2(x) = f_4(x) = ... = f_{2018}(x) = x$
and $f(x) = f_3(x) = ... = f_{2019}(x) = \frac{3x + 5}{2x - 3}$

$\therefore f_{2018}(x) + f_{2019}(x) = x + \frac{3x + 5}{2x - 3} = \frac{2x^2 + 5}{2x - 3}$

13. (a) : Given that $f : [1, \infty) \to [2, \infty)$, $f(x) = x + \frac{1}{x}$

Let $y \in [2, \infty)$

$\therefore f(x) = y \Rightarrow x + \frac{1}{x} = y$

$\Rightarrow x^2 - yx + 1 = 0 \Rightarrow x = \frac{y \pm \sqrt{y^2 - 4}}{2}$

Clearly $\forall y \in [2, \infty)$ there exist an unique x in $[1, \infty)$.
Thus f is onto.
Clearly, for $x_1, x_2 \in [1, \infty)$, $f(x_1) = f(x_2)$

$\Rightarrow x_1 + \frac{1}{x_1} = x_2 + \frac{1}{x_2} \Rightarrow (x_1 - x_2)\left(1 - \frac{1}{x_1 x_2}\right) = 0$

$\Rightarrow x_1 = x_2 \left[\because 1 - \frac{1}{x_1 x_2} \ne 0\right]$

So, f is one-one function.
Thus, f is one-one and onto, hence f is bijective.
Since, f is bijective, so it is invertible.

$\therefore f^{-1}(y) = x = \frac{y \pm \sqrt{y^2 - 4}}{2}$

or $f^{-1}(x) = \frac{x \pm \sqrt{x^2 - 4}}{2}$

Now, $f^{-1}(\sqrt{5}) = \frac{\sqrt{5} - 1}{2} < 1$ if negative sign is used

$\therefore f^{-1}(x) = \frac{x + \sqrt{x^2 - 4}}{2}$

14. (a) : The range of $|\sin x| = (0, 1]$
Now, A = range of $f(g(x))$
= range of $\log|\sin x| = (-\infty, 0]$
The range of $\log|x| = (-\infty, \infty)$
B = range of $g(f(x))$
= range of $\sin(\log|x|) = [-1, 1]$
$\therefore \quad A \cup B = (-\infty, 0] \cup [-1, 1] = (-\infty, 1]$
and $A \cap B = (-\infty, 0] \cap [-1, 1] = [-1, 0]$

15. (c) : Here, $f(x) = \sqrt{\dfrac{4-x^2}{2+[x]}}$

The function $f(x)$ will be defined if $\dfrac{4-x^2}{2+[x]} \geq 0$

Case I : $(4 - x^2) \geq 0$ and $2 + [x] > 0$
$\Rightarrow (x + 2)(x - 2) \leq 0$ and $[x] > -2$
$\Rightarrow x \in [-2, 2]$ and $x \in [-1, \infty)$
$\therefore \quad x \in [-1, 2]$ ... (i)
Case II : $4 - x^2 < 0$ and $2 + [x] < 0$
$\Rightarrow x \in R - [-2, 2]$ and $x \in (-\infty, -2)$
$\Rightarrow x \in (-\infty, -2)$ ...(ii)
From (i) and (ii), $x \in (-\infty, -2) \cup [-1, 2]$, which is the required domain.

16. (c) : We have, $f(x) = \tan^{-1}\dfrac{2x}{1-x^2} = 2\tan^{-1}x$

$\therefore \quad B = f(-1, 1) = \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

17. (b) : Here $f(x) = \sin x + \cos x$ and $g(x) = x^2 - 1$
$\therefore \quad g(f(x)) = (f(x))^2 - 1 = (\sin x + \cos x)^2 - 1 = \sin 2x$
We know that $\sin x$ is invertible in $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$\therefore \quad \sin 2x$ is invertibe in $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

18. (c) : Here, $f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$

$\Rightarrow \quad f(x) = 2\left(\dfrac{\sqrt{3}}{2}\sin 2x - \dfrac{1}{2}\cos 2x\right) + 4$

$\Rightarrow \quad f(x) = 2\sin\left(2x - \dfrac{\pi}{6}\right) + 4$

We know that $\sin x$ is one-one in $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$\therefore \quad -\dfrac{\pi}{2} \leq 2x - \dfrac{\pi}{6} \leq \dfrac{\pi}{2} \quad \Rightarrow \quad -\dfrac{\pi}{3} \leq 2x \leq \dfrac{2\pi}{3}$

$\Rightarrow \quad -\dfrac{\pi}{6} \leq x \leq \dfrac{\pi}{3} \quad \Rightarrow \quad x \in \left[-\dfrac{\pi}{6}, \dfrac{\pi}{3}\right]$

19. (d) : Here, $f(x) = \dfrac{x-[x]}{1-[x]+x} = \dfrac{x-[x]}{1+x-[x]}$

$\therefore \quad f(x) = \dfrac{\{x\}}{1+\{x\}}$

Since, $0 \leq \{x\} < 1$
$\therefore \quad 1 \leq 1 + \{x\} < 2$
Now, $1 + \{x\} = 1$ when $\{x\} = 0$
Thus, $f(x) \in \left[0, \dfrac{1}{2}\right]$

20. (d) : Period of $\sin\dfrac{\theta}{3} = \dfrac{2\pi}{1/3} = 6\pi$

and period of $\cos\dfrac{\theta}{2} = \dfrac{2\pi}{1/2} = 4\pi$

$\therefore \quad$ Period of $f(\theta)$ = L.C.M. $(6\pi, 4\pi) = 12\pi$.

21. (c) : Given $[\sin x] + [\sqrt{2}\cos x] = -3$
$\therefore \quad [\sin x] = -1$ and $[\sqrt{2}\cos x] = -2$
Now, $[\sin x] = -1$
$\Rightarrow \quad -1 \leq \sin x < 0$
$\Rightarrow \quad x \in (\pi, 2\pi)$ ... (i)
Again, $[\sqrt{2}\cos x] = -2 \quad \Rightarrow \quad -2 \leq \sqrt{2}\cos x < -1$

$\Rightarrow \quad -\sqrt{2} \leq \cos x < -\dfrac{1}{\sqrt{2}} \quad \Rightarrow \quad \cos x < -\dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad x \in \left(\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right)$ ... (ii)

From (i) and (ii), we get $x \in \left(\pi, \dfrac{5\pi}{4}\right)$

22. (a) : Given $\tan(x - y) = 1$ and $\sec(x + y) = \dfrac{2}{\sqrt{3}}$

When, $\tan(x - y) = 1$, then $x - y = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

When, $\sec(x + y) = \dfrac{2}{\sqrt{3}}$, then $x + y = \dfrac{\pi}{6}, \dfrac{11\pi}{6}$

Now, $x + y > x - y$ (x, y are positive)

We take $x + y = \dfrac{11\pi}{6}$ and $x - y = \dfrac{5\pi}{4}$

Solving, we get $x = \dfrac{37\pi}{24}$, $y = \dfrac{7\pi}{24}$

23. (c) : $e^{(\sin^2 x + \sin^4 x + \sin^6 x + \ldots \text{to } \infty)\log_e 2}$

$= e^{\left(\frac{\sin^2 x}{1-\sin^2 x}\right)\log_e 2}$

$= e^{\tan^2 x \log_e 2} = e^{\log_e 2^{\tan^2 x}} = 2^{\tan^2 x}$

Now, $x^2 - 9x + 8 = 0 \Rightarrow x = 1$ and $x = 8$

$\therefore \; 2^{\tan^2 x} = 1$ or $8 \Rightarrow 2^{\tan^2 x} = 2^0$ or 2^3

If $\tan^2 x = 0 \Rightarrow x = 0$

or $\tan^2 x = 3 \Rightarrow x = \pm\frac{\pi}{3}$

But only $\frac{\pi}{3}$ belongs to $0 < x < \frac{\pi}{2} \;\therefore\; x = \frac{\pi}{3}$.

24. (a) : Given $\theta = \frac{\pi}{4n} \;\therefore\; 2n\theta = \frac{\pi}{2}$

Now, $\tan(2n-1)\theta = \tan(2n\theta - \theta)$

$= \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

Also, $\tan(2n-2)\theta = \tan(2n\theta - 2\theta)$

$= \tan\left(\frac{\pi}{2} - 2\theta\right) = \cot 2\theta$

and so on.

Now, $\tan\theta \tan 2\theta \tan 3\theta \ldots \tan(2n-1)\theta$

$= \tan\theta \tan 2\theta \tan 3\theta \ldots \tan(2n-3)\theta \tan(2n-2)\theta \times \tan(2n-1)\theta$

$= \tan\theta \tan 2\theta \tan 3\theta \ldots \cot 3\theta \cot 2\theta \cot\theta$

$= (\tan\theta \cot\theta)(\tan 2\theta \cot 2\theta)(\tan 3\theta \cot 3\theta) \ldots$

$= 1 \cdot 1 \cdot 1 \ldots = 1$

25. (b) : We have $\sqrt{\frac{1}{\cos^2\alpha} - \frac{2}{\cot\alpha}} = \sqrt{\sec^2\alpha - 2\tan\alpha}$

$= \sqrt{1 + \tan^2\alpha - 2\tan\alpha} = \sqrt{(1-\tan\alpha)^2}$

$= |1 - \tan\alpha|$

$= \tan\alpha - 1 \left(\because \tan\alpha > 1 \text{ when } \alpha \in \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]\right)$

26. (a) : Here, the transformation is

$x = x' - 1$ and $y = y' + 2$

The given equation becomes

$2(x'-1)^2 + (y'+2)^2 - 4(x'-1) + 4(y'+2) = 0$

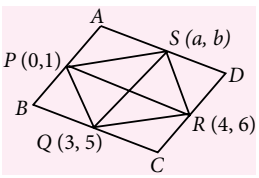
$\Rightarrow 2x'^2 + y'^2 - 8x' + 8y' + 18 = 0$

or $2x^2 + y^2 - 8x + 8y + 18 = 0$,

which is the required equation.

27. (c) : Clearly, P, Q, R and S are the vertices of parallelogram $PQRS$.

We know diagonals of a parallelogram bisect each other.

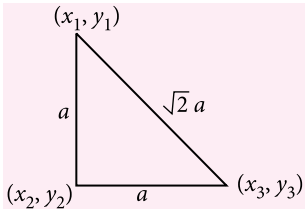


Hence, $\left(\frac{0+4}{2}, \frac{1+6}{2}\right) \equiv \left(\frac{3+a}{2}, \frac{5+b}{2}\right)$

$\Rightarrow \frac{3+a}{2} = 2 \Rightarrow a = 1$

and $\frac{5+b}{2} = \frac{7}{2} \Rightarrow b = 2$

28. (b) : Since, a is an integer, $\sqrt{2}a$ is irrational number.



Incentre will be $\left(\frac{ax_1 + ax_2 + \sqrt{2}ax_3}{2a + a\sqrt{2}}, \frac{ay_1 + ay_2 + \sqrt{2}ay_3}{2a + a\sqrt{2}}\right)$,

which is clearly an irrational point.

29. (c) : Let $A(x, y)$, $B(3, 4)$ and $C(-2, 3)$

Given that ΔABC is an equilateral triangle.

$\therefore \; AB = AC = BC$

Now $AB = AC$

$\Rightarrow AB^2 = AC^2$

$\Rightarrow (x-3)^2 + (y-4)^2 = (x+2)^2 + (y-3)^2$

$\Rightarrow 5x + y = 6$ or $y = 6 - 5x$ ...(i)

Again, $AC = BC$ gives

$(x+2)^2 + (y-3)^2 = 26$

$\Rightarrow (x+2)^2 + (3-5x)^2 = 26$ [Using (i)]

$\Rightarrow 2x^2 - 2x - 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{3}}{2}$

Now from (i), we get $y = \frac{7 \mp 5\sqrt{3}}{2}$

30. (c) : We know that, if vertices of a ΔABC are $\left(a, \frac{1}{a}\right)$, $\left(b, \frac{1}{b}\right)$, $\left(c, \frac{1}{c}\right)$, then the co-ordinate of orthocentre is $\left(-\frac{1}{abc}, -abc\right)$.

MPP-2 CLASS XII ANSWER KEY

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(b)
6.	(a)	**7.**	(b, c)	**8.**	(a, b, c)	**9.**	(a, b, c)		
10.	(a, c)	**11.**	(a, b, c)			**12.**	(b)	**13.**	(a, b, c)
14.	(c)	**15.**	(b)	**16.**	(d)	**17.**	(1)	**18.**	(1)
19.	(3)	**20.**	(0)						

SOLVED PAPER **2018**

Karnataka CET

1. For the LPP, maximise $z = x + 4y$ subject to the constraints $x + 2y \leq 2$, $x + 2y \geq 8$, $x, y \geq 0$.
(a) $z_{max} = 4$ (b) $z_{max} = 8$
(c) $z_{max} = 16$ (d) Has no feasible solution

2. For the probability distribution given by

$X = x_i$	0	1	2
P_i	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

the standard deviation (σ) is
(a) $\sqrt{\frac{1}{3}}$ (b) $\frac{1}{3}\sqrt{\frac{5}{2}}$
(c) $\sqrt{\frac{5}{36}}$ (d) None of the above

3. A bag contains 17 tickets numbered from 1 to 17. A ticket is drawn at random, then another ticket is drawn without replacing the first one. The probability that both the tickets may show even numbers is
(a) $\frac{7}{34}$ (b) $\frac{8}{17}$ (c) $\frac{7}{16}$ (d) $\frac{7}{17}$

4. A flashlight has 10 batteries out of which 4 are dead. If 3 batteries are selected without replacement and tested, then the probability that all 3 are dead is
(a) $\frac{1}{30}$ (b) $\frac{2}{8}$ (c) $\frac{1}{15}$ (d) $\frac{1}{10}$

5. If $|x + 5| \geq 10$, then
(a) $x \in (-15, 5]$ (b) $x \in (-5, 5]$
(c) $x \in (-\infty, -15] \cup [5, \infty)$
(d) $x \in [-\infty, -15] \cup [5, \infty)$

6. Everybody in a room shakes hands with everybody else. The total number of handshakes is 45. The total number of persons in the room is
(a) 9 (b) 10 (c) 5 (d) 15

7. The constant term in the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{16}$ is
(a) ${}^{16}C_8$ (b) ${}^{16}C_7$
(c) ${}^{16}C_9$ (d) ${}^{16}C_{10}$

8. If $P(n)$: "$2^{2n} - 1$ is divisible by k for all $n \in N$" is true, then the value of 'k' is
(a) 6 (b) 3 (c) 7 (d) 2

9. The equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through $(-2, 3)$ is
(a) $3x - 4y + 18 = 0$
(b) $3x - 4y - 18 = 0$
(c) $3x + 4y + 18 = 0$
(d) $3x + 4y - 18 = 0$

10. If $\left(\frac{1-i}{1+i}\right)^{96} = a + ib$ then (a, b) is
(a) $(1, 1)$ (b) $(1, 0)$
(c) $(0, 1)$ (d) $(0, -1)$

11. The distance between the focii of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is
(a) $x^2 - y^2 = 32$ (b) $\frac{x^2}{4} - \frac{y^2}{9} = 1$
(c) $2x^2 - 3y^2 = 7$ (d) $y^2 - x^2 = 32$

12. The number of ways in which 5 girls and 3 boys can be seated in a row so that no two boys are together is
(a) 14040 (b) 14440
(c) 14000 (d) 14400

13. If a, b, c are three consecutive terms of an A.P. and x, y, z are three consecutive terms of a G.P., then the value of $x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$ is
(a) 0 (b) xyz
(c) -1 (d) 1

14. The value of $\lim_{x\to 0}\frac{|x|}{x}$ is

(a) 1 (b) −1
(c) 0 (d) Does not exist

15. Let $f(x)=x-\frac{1}{x}$ then $f'(-1)$ is

(a) 0 (b) 2 (c) 1 (d) −2

16. The negation of the statement "72 is divisible by 2 and 3" is

(a) 72 is not divisible by 2 or 72 is not divisible by 3
(b) 72 is divisible by 2 or 72 is divisible by 3
(c) 72 is divisible by 2 and 72 is divisible by 3
(d) 72 is not divisible by 2 and 3

17. The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B is

(a) 0.4 (b) 0.5 (c) 0.2 (d) 0.9

18. In a simultaneous throw of a pair of dice, the probability of getting a total more than 7 is

(a) $\frac{7}{12}$ (b) $\frac{5}{36}$ (c) $\frac{5}{12}$ (d) $\frac{7}{36}$

19. If A and B are mutually exclusive events, given that $P(A)=\frac{3}{5}$, $P(B)=\frac{1}{5}$, then $P(A \text{ or } B)$ is

(a) 0.8 (b) 0.6 (c) 0.4 (d) 0.2

20. Let $f, g : R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x \ \forall\ x \in R$. Then $(fog)(x)$ for $x < 0$ is

(a) 0 (b) $4x$ (c) $-4x$ (d) $2x$

21. A is a set having 6 distinct elements. The number of distinct functions from A to A which are not bijections is

(a) $6! - 6$ (b) $6^6 - 6$
(c) $6^6 - 6!$ (d) $6!$

22. Let $f: R \to R$ be defined by

$$f(x)=\begin{cases}2x & ; \quad x>3\\ x^2 & ; \quad 1<x\le 3\\ 3x & ; \quad x\le 1\end{cases}$$

Then $f(-1) + f(2) + f(4)$ is

(a) 9 (b) 14 (c) 5 (d) 10

23. If $\sin^{-1}x+\cos^{-1}y=\frac{2\pi}{5}$, then $\cos^{-1}x + \sin^{-1}y$ is

(a) $\frac{2\pi}{5}$ (b) $\frac{3\pi}{5}$ (c) $\frac{4\pi}{5}$ (d) $\frac{3\pi}{10}$

24. The value of the expression $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$ is

(a) $2-\sqrt{5}$ (b) $\sqrt{5}-2$
(c) $\frac{\sqrt{5}-2}{2}$ (d) $5-\sqrt{2}$

25. If $A=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$, then $A^n = 2^kA$, where $k =$

(a) 2^{n-1} (b) $n + 1$
(c) $n - 1$ (d) $2(n - 1)$

26. If $\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}2\\ 4\end{bmatrix}$ then the values of x and y respectively are

(a) −3, −1 (b) 1, 3
(c) 3, 1 (d) −1, 3

27. If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then $AA' =$

(a) A (b) zero matrix
(c) A' (d) I

28. If $x, y, z \in R$, then the value of the determinant

$$A=\begin{bmatrix}(5^x+5^{-x})^2 & (5^x-5^{-x})^2 & 1\\ (6^x+6^{-x})^2 & (6^x-6^{-x})^2 & 1\\ (7^x+7^{-x})^2 & (7^x-7^{-x})^2 & 1\end{bmatrix} \text{ is}$$

(a) 10 (b) 12 (c) 1 (d) 0

29. The value of determinant $\begin{vmatrix}a-b & b+c & a\\ b-a & c+a & b\\ c-a & a+b & c\end{vmatrix}$ is

(a) $a^3 + b^3 + c^3$ (b) $3abc$
(c) $a^3 + b^3 + c^3 - 3abc$
(d) $a^3 + b^3 + c^3 + 3abc$

30. If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of a triangle whose area is k square units, then

$$\begin{vmatrix}x_1 & y_1 & 4\\ x_2 & y_2 & 4\\ x_3 & y_3 & 4\end{vmatrix}^2 \text{ is}$$

(a) $32k^2$ (b) $16k^2$
(c) $64k^2$ (d) $48k^2$

31. Let A be a square matrix of order 3×3, then $|5A| =$

(a) $5|A|$ (b) $125|A|$
(c) $25|A|$ (d) $15|A|$

32. If $f(x)=\begin{cases}\dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x} & \text{if} \quad -1\le x<0\\ \dfrac{2x+1}{x-1} & \text{if} \quad 0\le x\le 1\end{cases}$ is continuous at $x = 0$, then the value of k is

(a) $k = 1$ (b) $k = -1$
(c) $k = 0$ (d) $k = 2$

33. If $\cos y = x\cos(a + y)$ with $\cos a \neq \pm 1$, then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{\sin a}{\cos^2(a+y)}$ (b) $\dfrac{\cos^2(a+y)}{\sin a}$

(c) $\dfrac{\cos a}{\sin^2(a+y)}$ (d) $\dfrac{\cos^2(a+y)}{\cos a}$

34. If $f(x) = |\cos x - \sin x|$, then $f'(\pi/6)$ is equal to

(a) $-\dfrac{1}{2}(1+\sqrt{3})$ (b) $\dfrac{1}{2}(1+\sqrt{3})$

(c) $-\dfrac{1}{2}(1-\sqrt{3})$ (d) $\dfrac{1}{2}(1-\sqrt{3})$

35. If $y=\sqrt{x+\sqrt{x+\sqrt{x+....\infty}}}$, then $\dfrac{dy}{dx}=$

(a) $\dfrac{1}{y^2-1}$ (b) $\dfrac{1}{2y+1}$

(c) $\dfrac{2y}{y^2-1}$ (d) $\dfrac{1}{2y-1}$

36. If $f(x)=\begin{cases}\dfrac{\log_e x}{x-1} & ; \quad x\neq 1\\ k & ; \quad x=1\end{cases}$ is

continuous at $x = 1$, then the value of k is

(a) e (b) 1 (c) -1 (d) 0

37. Approximate change in the volume V of a cube of side x metres caused by increasing the side by 3% is

(a) $0.09x^3$ m^3 (b) $0.03x^3$ m^3
(c) $0.06x^3$ m^3 (d) $0.04x^3$ m^3

38. The maximum value of $\left(\dfrac{1}{x}\right)^x$ is

(a) e (b) e^e (c) $e^{1/e}$ (d) $\left(\dfrac{1}{e}\right)^{1/e}$

39. $f(x) = x^x$ has stationary point at

(a) $x = e$ (b) $x=\dfrac{1}{e}$

(c) $x = 1$ (d) $x=\sqrt{e}$

40. The maximum area of a rectangle inscribed in the circle $(x + 1)^2 + (y - 3)^2 = 64$ is

(a) 64 sq. units (b) 72 sq. units
(c) 128 sq. units (d) 8 sq. units

41. $\displaystyle\int \frac{1}{1+e^x}dx$ is equal to

(a) $\log_e\left(\dfrac{e^x+1}{e^x}\right)+c$ (b) $\log_e\left(\dfrac{e^x-1}{e^x}\right)+c$

(c) $\log_e\left(\dfrac{e^x}{e^x+1}\right)+c$ (d) $\log_e\left(\dfrac{e^x}{e^x-1}\right)+c$

42. $\displaystyle\int \frac{1}{\sqrt{3-6x-9x^2}}dx$ is equal to

(a) $\sin^{-1}\left(\dfrac{3x+1}{2}\right)+c$

(b) $\sin^{-1}\left(\dfrac{3x+1}{6}\right)+c$

(c) $\dfrac{1}{3}\sin^{-1}\left(\dfrac{3x+1}{2}\right)+c$

(d) $\sin^{-1}\left(\dfrac{2x+1}{3}\right)+c$

43. $\displaystyle\int e^{\sin x}\cdot\left(\frac{\sin x+1}{\sec x}\right)dx$ is equal to

(a) $\sin x \cdot e^{\sin x} + c$ (b) $\cos x \cdot e^{\sin x} + c$
(c) $e^{\sin x} + c$ (d) $e^{\sin x}(\sin x + 1) + c$

44. $\displaystyle\int_{-2}^{2} |x\cos \pi x|\,dx$ is equal to

(a) $\dfrac{8}{\pi}$ (b) $\dfrac{4}{\pi}$ (c) $\dfrac{2}{\pi}$ (d) $\dfrac{1}{\pi}$

45. $\displaystyle\int_0^1 \frac{dx}{e^x+e^{-x}}$ is equal to

(a) $\dfrac{\pi}{4}-\tan^{-1}(e)$ (b) $\tan^{-1}(e)-\dfrac{\pi}{4}$

(c) $\tan^{-1}(e)+\dfrac{\pi}{4}$ (d) $\tan^{-1}(e)$

46. $\displaystyle\int_0^{1/2} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ is equal to

(a) $\dfrac{1}{\sqrt{2}}\tan^{-1}\sqrt{\dfrac{2}{3}}$ (b) $\dfrac{2}{\sqrt{2}}\tan^{-1}\left(\dfrac{3}{\sqrt{2}}\right)$

(c) $\dfrac{\sqrt{2}}{2}\tan^{-1}\left(\dfrac{3}{2}\right)$ (d) $\dfrac{\sqrt{2}}{2}\tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

47. The area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is
(a) 1 sq. unit (b) 4 sq. units
(c) 2 sq. units (d) 3 sq. units

48. The area bounded by the line $y = x$, x-axis and ordinates $x = -1$ and $x = 2$ is
(a) 3/2 sq. units (b) 5/2 sq. units
(c) 2 sq. units (d) 3 sq. units

49. The degree and order of the differential equation $\frac{d^2y}{dx^2} = \sqrt[3]{1+\left(\frac{dy}{dx}\right)^2}$ respectively are
(a) 2 and 3 (b) 3 and 2
(c) 2 and 2 (d) 3 and 3

50. The solution of the differential equation $x\frac{dy}{dx} - y = 3$ represents a family of
(a) straight lines (b) circles
(c) parabolas (d) ellipses

51. The integrating factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is
(a) xe^x (b) $xe^{1/x}$ (c) $\frac{e^x}{x}$ (d) $\frac{x}{e^x}$

52. If $|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then the value of $|\vec{b}|$ is
(a) 1 (b) 2 (c) 3 (d) 4

53. If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors, then $(3\vec{a}+2\vec{b})\cdot(5\vec{a}-6\vec{b}) =$
(a) 5 (b) 3 (c) 6 (d) 12

54. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar ($a \neq b \neq c \neq 1$), then the value of $abc - (a + b + c) =$
(a) 2 (b) −2 (c) 0 (d) −1

55. If $\vec{a} = \hat{i}+\lambda\hat{j}+2\hat{k}$; $\vec{b} = \mu\hat{i}+\hat{j}-\hat{k}$ are orthogonal and $|\vec{a}| = |\vec{b}|$, then $(\lambda, \mu) =$
(a) $\left(\frac{1}{4}, \frac{7}{4}\right)$ (b) $\left(\frac{7}{4}, \frac{1}{4}\right)$
(c) $\left(\frac{1}{4}, \frac{9}{4}\right)$ (d) $\left(\frac{-1}{4}, \frac{9}{4}\right)$

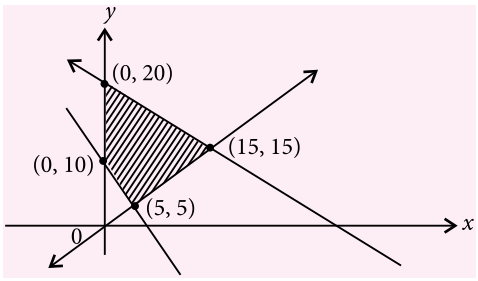
56. The image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is
(a) (1, 0, 7) (b) (7, 0, 1)
(c) (2, 7, 0) (d) (−1, −6, −3)

57. The angle beween the lines $2x = 3y = -z$ and $6x = -y = -4z$ is
(a) 0° (b) 45° (c) 90° (d) 30°

58. The value of k such that the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies on the plane $2x - 4y + z = 7$ is
(a) −7 (b) 4 (c) −4 (d) 7

59. The locus represented by $xy + yz = 0$ is
(a) a pair of perpendicular lines
(b) a pair of parallel lines
(c) a pair of parallel planes
(d) a pair of perpendicular planes

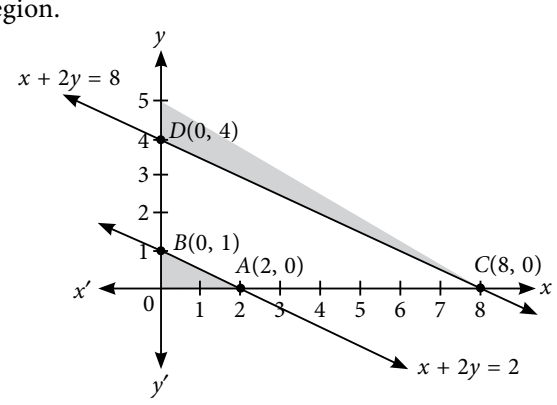
60. The feasible region of an LPP is shown in the figure. If $z = 3x + 9y$, then the minimum value of z occurs at



(a) (5, 5) (b) (0, 10)
(c) (0,20) (d) (15, 15)

SOLUTIONS

1. (d): Converting inequations into equations and drawing the corresponding lines.
$x + 2y = 2$, $x + 2y = 8$
i.e., $\frac{x}{2}+\frac{y}{1}=1$, $\frac{x}{8}+\frac{y}{4}=1$
$\because$ $x \geq 0, y \geq 0$
Since, there is no common region. So, it has no feasible region.



VK.COM/CLUB154894262

2. (b) :

x_i	P_i	P_ix_i	$P_ix_i^2$
0	$\frac{25}{36}$	0	0
1	$\frac{5}{18}$	$\frac{5}{18}$	$\frac{5}{18}$
2	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{9}$
		$\sum P_ix_i = \frac{1}{3}$	$\sum P_ix_i^2 = \frac{7}{18}$

Variance, $\sigma^2 = \sum P_ix_i^2 - (\Sigma P_ix_i)^2 = \frac{7}{18} - \frac{1}{9} = \frac{5}{18}$

Standard deviation, $\sigma = \sqrt{\frac{5}{18}} = \frac{1}{3}\sqrt{\frac{5}{2}}$

3. (a) : Out of 17 tickets in the bag one ticket can be drawn in 17 ways.
Now, out of 8 even numbered tickets, namely, 2, 4, 6, 8, 10, 12, 14, 16 one can be drawn in 8 ways.
$\because$ Tickets are drawn without replacing
So, we have to select 7 even numbered tickets out of remaining 16.

$\therefore$ Requred probability = $\frac{8}{17} \times \frac{7}{16} = \frac{7}{34}$

4. (a) : Possible outcomes of drawing 3 batteries out of 10 = $^{10}C_3$.
Number of dead batteries = 4

$\therefore$ P(drawing 3 dead batteries) = $\frac{^4C_3}{^{10}C_3} = \frac{4! \times 7!}{10!} = \frac{1}{30}$

5. (c) : We know that $|x - a| \geq r \Leftrightarrow x \leq a - r$,
or $x \geq a + r$
$\therefore$ $|x + 5| \geq 10 \Leftrightarrow x \leq -5 - 10$, or $x \geq -5 + 10$
$\Leftrightarrow$ $x \leq -15$ or $x \geq 5$
$\Leftrightarrow$ $x \in (-\infty, -15]$ or $x \in [5, \infty)$
$\Leftrightarrow$ $x \in (-\infty, -15] \cup [5, \infty)$

6. (b) : Let total number persons in the room = x
$\because$ Total numer of handshakes = 45

$\therefore$ $^xC_2 = 45 \Rightarrow \frac{x(x-1)}{2 \cdot 1} = 45$

$\Rightarrow$ $x^2 - x = 90 \Rightarrow x^2 - x - 90 = 0$
$\Rightarrow$ $(x + 9)(x - 10) = 0$
$\Rightarrow$ $x = 10$ ($\because$ $x = -9$ is not possible).
Hence, number of persons = 10

7. (a) : Let $(r + 1)^{th}$ term be the constant term in the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{16}$

$\therefore$ $T_{r+1} = {}^{16}C_r x^{2r} \left(-\frac{1}{x^2}\right)^{16-r}$

$= (-1)^{16-r} \cdot {}^{16}C_r \, x^{2r} \cdot x^{-32+2r}$
$= (-1)^{16-r} \cdot {}^{16}C_r \, x^{4r-32}$
For constant term, $4r - 32 = 0 \Rightarrow r = 8$
$\therefore$ $T_9 = (-1)^8 \, {}^{16}C_8 = {}^{16}C_8$

8. (b) : Let $P(n) = 2^{2n} - 1$ is divisible by k for all $n \in N$
$\therefore$ $P(1) = 2^2 - 1$ is divisible k
$\Rightarrow$ 3 is divisible by k
So, the value of $k = 3$

9. (a) : Given line is $3x - 4y + 2 = 0$ ...(i)
Equation of the line parallel to the (i) is $3x - 4y + k = 0$ ...(ii)
$\therefore$ Line (ii) passes through the point $(-2, 3)$
$\therefore$ $3(-2) - (4 \times 3) + k = 0 \Rightarrow -6 - 12 + k = 0$
$\Rightarrow$ $k = 18$
$\therefore$ $3x - 4y + 18 = 0$ is the required equation of line.

10. (b) : Given, $\frac{1-i}{1+i} = \frac{1-i}{1+i} \times \frac{1-i}{1-i} = \frac{(1-i)^2}{1-i^2} = -i$

$\therefore$ $\left(\frac{1-i}{1+i}\right)^{96} = (-i)^{96} = 1 = 1 + 0i$ ...(i)

Comparing (i) with $a + ib$, we get $a = 1$, $b = 0$
So, the value of $(a, b) = (1, 0)$

11. (a) : We have, $2ae = 16$ ...(i) and $e = \sqrt{2}$ ...(ii)
From (i) and (ii) we have,

$a = \frac{16}{2\sqrt{2}} = 4\sqrt{2} \Rightarrow a^2 = 32$

Since, $e = \sqrt{2}$ $\Rightarrow$ it is a rectangular hyperbola
Hence, the equation of hyperbola $\Rightarrow x^2 - y^2 = 32$

12. (d) : We have to find ways such that no two boys are together among 5 girls and 3 boys.
First we arrange 5 girls. This can be done in G ways.
$\circ$ G $\circ$ G $\circ$ G $\circ$ G $\circ$ G $\circ$
Now, there are 6 places which are to be filled by 3 boys. This can be done in $^6C_3$ places. Also, 3 boys can arrange themselves in 3! ways.
$\therefore$ Required ways = $5! \times {}^6C_3 \times 3!$ = 14,400

13. (d) : Given, $a, b, c, \in$ A.P $\Rightarrow$ $a + c = 2b$ ...(i)
Also, $x, y, z \in$ G.P.
Let r be the common ratio of the G.P.
$\therefore$ $x = x$, $y = xr$, $z = xr^2$ ...(ii)
Now, $x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$

SERIAL BOOKS AND MAGAZINES

$= x^{b-c}.\ (xr)^{c-a}\ .\ (xr^2)^{a-b}$ [Using (ii)]

$= x^{b-c+c-a+a-b}\ .\ r^{c-a+2a-2b}$

$= x^0.\ r^{a-2b+c} = 1\cdot r^{2b-2b}$ [Using (i)]

$= 1\cdot 1 = 1$

14. (d) : L.H.L. $= \lim\limits_{x\to 0^-} \dfrac{|x|}{x} = \lim\limits_{x\to 0^-} \dfrac{-x}{x} = \lim\limits_{x\to 0^-} -1 = -1$

R.H.L. $= \lim\limits_{x\to 0^+} \dfrac{|x|}{x} = \lim\limits_{x\to 0^+} \dfrac{x}{x} = \lim\limits_{x\to 0^+} 1 = 1$

Hence, $\lim\limits_{x\to 0} \dfrac{|x|}{x}$ does not exist.

15. (b) : Given, $f(x) = x - \dfrac{1}{x}$

$\Rightarrow\ f'(x) = 1 + \dfrac{1}{x^2}$

$f'(-1) = 1 + \dfrac{1}{(-1)^2} = 1 + 1 = 2$

16. (a) : The negation of the statement "72 is divisible by 2 and 3" is "72 is not divisible by 2 or 72 is not divisible by 3."

17. (c) : $\because$ A and B are mutually exclusive events.

$\therefore\ P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.3 = 0.8$

Thus, $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

18. (c) : Let E be the event that the sum of numbers which come up on the dice is more than 7.

E = {(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore\ n(E) = 15$

Total number of outcomes $= 6 \times 6 = 36$

So, Required probability $= \dfrac{15}{36} = \dfrac{5}{12}$

19. (a) : $\because$ Given, A and B are mutually exclusive events

$\therefore\ P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{3}{5} + \dfrac{1}{5} = \dfrac{4}{5} = 0.8$

20. (c) : $(fog)(x) = f(g(x)) = f(|x| - x)$

$= ||x| - x| + |x| - x = |x| - x + |x| - x = -4x$

21. (c) : Total number of functions from A to $A = 6^6$

number of bijective function = 6!

$\therefore$ Number of functions which are not bijective $= 6^6 - 6!$

22. (a) : We have, $f(-1) + f(2) + f(4)$

$= 3(-1) + (2)^2 + 2(4) = -3 + 4 + 8 = 9$

23. (b) : We have, $\sin^{-1} x + \cos^{-1} y = \dfrac{2\pi}{5}$

$\Rightarrow\ \dfrac{\pi}{2} - \cos^{-1} x + \dfrac{\pi}{2} - \sin^{-1} y = \dfrac{2\pi}{5}$

$\left\{\text{Using } \sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}\right\}$

$\Rightarrow\ \pi - \cos^{-1} x - \sin^{-1} y = \dfrac{2\pi}{5}$

$\Rightarrow\ \cos^{-1} x + \sin^{-1} y = \dfrac{3\pi}{5}$

24. (b) : Let $\tan\left(\dfrac{1}{2}\cos^{-1}\dfrac{2}{\sqrt{5}}\right) = \theta$

$\cos^{-1}\dfrac{2}{\sqrt{5}} = 2\tan^{-1}\theta\ \Rightarrow\ \cos^{-1}\dfrac{2}{\sqrt{5}} = \cos^{-1}\left(\dfrac{1-\theta^2}{1+\theta^2}\right)$

$\Rightarrow\ \dfrac{2}{\sqrt{5}} = \dfrac{1-\theta^2}{1+\theta^2}\ \Rightarrow\ 2 + 2\theta^2 = \sqrt{5} - \sqrt{5}\,\theta^2$

$\Rightarrow\ \theta^2(\sqrt{5} + 2) = \sqrt{5} - 2$

$\Rightarrow\ \theta^2 = \dfrac{\sqrt{5}-2}{\sqrt{5}+2}\ \Rightarrow\ \theta^2 = (\sqrt{5}-2)^2\ \Rightarrow\ \theta = \sqrt{5} - 2$

25. (d) : Given, $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$

$= 2^2\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2^2 A$ ...(i)

Similarly, $A^3 = A^2 \cdot A = 2^2 A$ [From (i)]

$= 2^2 \cdot 2^2 A = 2^4 A = 2^{2(3-1)} A$

$A^4 = A^2 \cdot A^2 = 2^2 A \cdot 2^2 \cdot A$

$= 2^4 A^2 = 2^6 A = 2^{2(4-1)} A$

Hence, $A^n = 2^{2(n-1)} A$, So, $k = 2(n - 1)$

26. (d) : We have, $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\Rightarrow\ x + y = 2$ ....(i)

and $-x + y = 4$ ....(ii)

Adding (i) and (ii) we get,

$2y = 6 \Rightarrow y = 3$

Putting value of y in (i), we get $x = -1$

27. (d) : We have, $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A \cdot A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

28. (d) : Let $\Delta = \begin{vmatrix} (5^x + 5^{-x})^2 & (5^x - 5^{-x})^2 & 1 \\ (6^x + 6^{-x})^2 & (6^x - 6^{-x})^2 & 1 \\ (7^x + 7^{-x})^2 & (7^x - 7^{-x})^2 & 1 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$, we get

$$\Delta = \begin{vmatrix} 4 & (5^x - 5^{-x})^2 & 1 \\ 4 & (6^x - 6^{-x})^2 & 1 \\ 4 & (7^x - 7^{-x})^2 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = 4\begin{vmatrix} 1 & (5^x - 5^{-x})^2 & 1 \\ 1 & (6^x - 6^{-x})^2 & 1 \\ 1 & (7^x - 7^{-x})^2 & 1 \end{vmatrix} = 0 \qquad (\because C_1 \sim C_3)$$

29. (None of the option is correct) :

Let $\Delta = \begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix}$

Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} -b & a+b+c & a \\ -a & a+b+c & b \\ -a & a+b+c & c \end{vmatrix}$$

Taking common $(a + b + c)$ from C_2, we get,

$$\Delta = (a+b+c)\begin{vmatrix} -b & 1 & a \\ -a & 1 & b \\ -a & 1 & c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Delta = (a+b+c)\begin{vmatrix} -b & 1 & a \\ b-a & 0 & b-a \\ b-a & 0 & c-a \end{vmatrix}$$

Expanding along C_2, we get

$\Delta = -(a + b + c)\,[(b - a)\,(c - a) - (b - a)^2]$

$= -(a + b + c)\,(b - a)\,[c - a - b + a]$

$= -(a + b + c)\,(b - a)\,(c - b)$

$= (a + b + c)\,(a - b)\,(c - b)$

30. (c) : Let k be the area of a triangle ABC. Then

$$k = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \Rightarrow 2k = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore \ 8k = 4\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \Rightarrow 8k = \begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}$$

$$\Rightarrow 64k^2 = \begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2$$

31. (b) : If A is a square matrix of order 3×3, then $|5A| = 5^3|A| = 125|A|$

32. (b) : Given f is continuous at $x = 0$

$$\text{Now, L.H.L} = \lim_{x\to 0}\frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}$$

$$= \lim_{x\to 0}\frac{2kx}{x[\sqrt{1+kx} + \sqrt{1-kx}]} = k$$

$$\text{R.H.L} = \lim_{x\to 0}\frac{2x+1}{x-1} = -1 \qquad \therefore\ k = -1$$

33. (b) : Given, $\cos y = x\cos(a + y)$

$$\Rightarrow \ x = \frac{\cos y}{\cos(a+y)}$$

On differentiating both sides w.r.t x, we get

$$1 = \frac{-\cos(a+y)\sin y\frac{dy}{dx} + \sin(a+y)\cos y\frac{dy}{dx}}{\cos^2(a+y)}$$

$$\cos^2(a + y) = \sin(a + y - y)\frac{dy}{dx}$$

$$\Rightarrow \ \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

34. (a) : We have $f(x) = |\cos x - \sin x|$

In the neighbourhood of $\frac{\pi}{6}$, we have $f(x)\ \cos x - \sin x$

$$\Rightarrow \ f'(x) = -\sin x - \cos x$$

$$\therefore \ f'\left(\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} - \cos\frac{\pi}{6} = \frac{-1}{2} - \frac{\sqrt{3}}{2} = -\frac{1}{2}[1 + \sqrt{3}]$$

MPP-2 CLASS XI ANSWER KEY

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(b)	**5.**	(c)
6.	(c)	**7.**	(a, b, c)			**8.**	(a, b, c)		
9.	(a, b, c, d)			**10.**	(a,b)	**11.**	(a, b, c)		
12.	(c)	**13.**	(b, c)	**14.**	(b)	**15.**	(a)	**16.**	(a)
17.	(1)	**18.**	(6)	**19.**	(1)	**20.**	(1)		

35. (d) : Given, $y=\sqrt{x+\sqrt{x+\sqrt{x+....\infty}}}$

$\Rightarrow \quad y=\sqrt{x+y} \quad \Rightarrow \quad y^2=x+y$ ...(i)

DIfferentiating (i) on both sides, we get

$$2y\frac{dy}{dx}=1+\frac{dy}{dx} \quad \Rightarrow \quad \frac{dy}{dx}=\frac{1}{2y-1}$$

36. (b) : For continuity at $x = 1$, $\lim\limits_{x\to 1} f(x)=f(1)$

$$\Rightarrow \quad \lim_{x\to 1}\frac{\log x}{x-1}=k \Rightarrow \lim_{x\to 1}\frac{1/x}{1}=k \quad \Rightarrow k=1$$

37. (a) : It is given that $\frac{\Delta x}{x}\times 100=3$ ...(i)

We have, $V=x^3 \Rightarrow \frac{dV}{dx}=3x^2$ ...(ii)

Now, $\Delta V=\frac{dV}{dx}\times\Delta x \Rightarrow \Delta V=3x^2\Delta x$ [Using (ii)]

$\Rightarrow \quad \Delta V=3x^2\times\frac{3x}{100}$ [Using (i)] $\Rightarrow \Delta V = 0.09x^3$ m^3

38. (c) : Let $y=\left(\frac{1}{x}\right)^x \Rightarrow \log y=x\log\frac{1}{x}$

$\Rightarrow$ log$y$ = – x log$x$

On differentiating both sides w.r.t x, we get

$$\frac{1}{y}\frac{dy}{dx}=-1-\log x \quad \Rightarrow \quad \frac{dy}{dx}=-y(1+\log x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}=-\frac{dy}{dx}(1+\log x)-\frac{y}{x}=y(1+\log x)^2-\frac{y}{x}$$

$$\frac{d^2y}{dx^2}=\left(\frac{1}{x}\right)^x(1+\log x)^2-\frac{1}{x^{(x+1)}}$$

Now, $\frac{dy}{dx}=0 \Rightarrow -y\,(1+\log x)=0$

$\Rightarrow \quad 1+\log x=0$ $(\because y\neq 0)$

$\Rightarrow \quad \log x=-1 \Rightarrow x=e^{-1} \Rightarrow x=1/e$

Also, $\left[\frac{d^2y}{dx^2}\right]_{x=\frac{1}{e}}=e^{\frac{1}{e}}\left(1+\log\frac{1}{e}\right)^2-e^{\left(\frac{1}{e}+1\right)}$

$$=e^{\frac{1}{e}}(1-\log e)^2-e^{\frac{1}{e}+1}=-e^{\frac{1}{e}+1}<0$$

So, $x = 1/e$ is a point of maxima.

Hence, maximum value of y is $e^{1/e}$

39. (b) : We have, $f(x) = x^x$

$\Rightarrow \quad f'(x)=x^x\,(1+\log x)$

for stationary point, $f'(x) = 0$

$\Rightarrow \quad x^x\,(1+\log x)=0 \Rightarrow 1+\log x=0$ $(\because x^x\neq 0)$

$\Rightarrow \quad \log x=-1 \Rightarrow x=e^{-1} \quad \therefore \quad x=\frac{1}{e}$

40. (c)

41. (c) : Let $I=\int\frac{1}{1+e^x}dx \Rightarrow I=\int\frac{e^{-x}}{e^{-x}+1}dx$

$\Rightarrow \quad I=-\int\frac{-e^{-x}}{e^{-x}+1}dx \Rightarrow I=-\log_e(e^{-x}+1)+c$

$$=\log_e\left(\frac{e^x}{1+e^x}\right)+c$$

42. (c) : We have,

$$\int\frac{dx}{\sqrt{(3-6x-9x^2)}}=\frac{1}{3}\int\frac{dx}{\sqrt{(-1)\left[x^2+\frac{2}{3}x-\frac{1}{3}\right]}}$$

$$=\frac{1}{3}\int\frac{dx}{\sqrt{(-1)\left[\left(x^2+\frac{2}{3}x+\frac{1}{9}\right)-\frac{4}{9}\right]}}$$

$$=\frac{1}{3}\int\frac{dx}{\sqrt{\left(\frac{2}{3}\right)^2-\left(x+\frac{1}{3}\right)^2}}$$

$$=\frac{1}{3}\sin^{-1}\left[\frac{\left(x+\frac{1}{3}\right)}{\frac{2}{3}}\right]+c=\frac{1}{3}\sin^{-1}\left(\frac{(3x+1)}{2}\right)+c$$

43. (a) : Let $I=\int e^{\sin x}\left(\frac{\sin x+1}{\sec x}\right)dx$

$$=\int e^{\sin x}(\sin x+1)\cos x\,dx$$

Put sin$x$ = t $\Rightarrow$ cos$x$ dx = dt

$$I=\int_{\text{II}}\underset{\text{I}}{e^t(t+1)}\,dt$$

$$=(t+1)\,e^t-\int 1\cdot e^t\,dt \;=e^t(t+1)-e^t+c$$

$$=te^t+c \;=\sin x\,e^{\sin x}+c$$

44. (a) 45. (b) 46. (a) 47. (c) 48. (b)
49. (b) 50. (a) 51. (c) 52. (c) 53. (b)
54. (b) 55. (a) 56. (a) 57. (c) 58. (d)
59. (d) 60. (a)

For complete solutions refer to MTG Karnataka CET Explorer.

Solution Sender of Maths Musing

SET-185

- Kilaparthi Thrinadha (Andhra Pradesh)
- Gouri Sankar Adhikary (West Bengal)
- N. Jayanthi (Hyderabad)
- Baibhab Chakraborty (Assam)

J&K CET

Jammu & Kashmir Common Entrance Test

SOLVED PAPER 2018

1. On a biased dice, numbers from 1 to 6 are written on its six faces. The probability that a face will appear on throwing the dice is directly proportional to number written on that face. What is the probability that a face on which a multiple of 3 is written will appear both times when this dice is thrown twice?
(a) 5/28 (b) 3/14 (c) 2/7 (d) 9/49

2. What will be the argument of the complex number $z = 1 - \cos p + i\sin p$?
(a) $\pi - p$ (b) $\pi/4 - p/2$
(c) $\pi/2 - p/2$ (d) $\pi - p/2$

3. The first order derivative of $f(x)$ is given by $f'(x) = \sec^4 x + 5$ with $f(0) = 0$. Then $f(x) =$
(a) $\tan x + \frac{1}{3}\tan^3 x + 5x + 2$
(b) $\tan x + \frac{1}{3}\tan^3 x + 5x$
(c) $\tan x + \frac{1}{3}\tan^3 x$ (d) $\frac{1}{3}\tan^3 x + 5x$

4. A plane is given as $3x + 4y - 12z + 13 = 0$. The points(1, 1, k) and (–3, 0, 1) are equidistant from the plane. What will be the quadratic equation that will have roots as values of k?
(a) $4x^2 - 15x + 9 = 0$ (b) $x^2 - 10x + 25 = 0$
(c) $3x^2 - 8x + 4 = 0$ (d) $3x^2 - 10x + 7 = 0$

5. The square of height of a tower is three times the square of its shadow. What would be the angle of depression of sun at this moment?
(a) 30° (b) 45° (c) 60° (d) 75°

6. If $[x]$ represents greatest integer less than or equal to x, then $\int_2^5 2[x]dx =$
(a) 9 (b) 12 (c) 18 (d) 24

7. Given that a function $f(x) = [x]^2 + x^2$, where $[x]$ is the greatest integer less than or equal to x. If $f(x) > 25$, then the value of x is
(a) any real number
(b) a member of the set $\{x | x \geq 0\}$
(c) a member of the set $\{x | x \leq -4 \text{ or } x \geq 4\}$
(d) a member of the set $\{x | x \geq 25 \text{ or } x \leq 0\}$

8. How many digits will be there after decimal point in 12^{th} term of the sequence: 32, 16, 8, 4, ... ?
(a) 5 (b) 6 (c) 7 (d) 8

9. From a deck of cards, all the diamonds are removed. From remaining cards, a card is chosen randomly. What is the probability that it will be a black card?
(a) 1/2 (b) 2/3 (c) 1/3 (d) 3/4

10. 4 consecutive years are taken such that exactly one of them is a leap year. One of these four years is randomly chosen. What is the probability that it will have 53 Wednesdays ?
(a) 1/7 (b) 3/14 (c) 1/4 (d) 5/28

11. $M = [m_{ij}]$, is a 4×4 square matrix, where $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$. In M, $m_{ij} = \sin(i \cdot j)\, x$. What will be the value of the determinant $|M|$ at $x = \pi/3$?
(a) 0 (b) 1 (c) –1 (d) 1/2

12. What will be the equation of the standard hyperbola where foci are (0, ±10) and the length of the latus rectum is 30?
(a) $3y^2 - x^2 = 75$ (b) $y^2 - x^2 = 25$
(c) $3y^2 - x^2 = 25$ (d) $y^2 - x^2 = 75$

13. It is given that $z_r = \cos(\pi/2^r) + i\sin(\pi/2^r)$. The value of the product $z_1 \cdot z_2 \cdot z_3 \cdot z_4 \ldots\ldots\infty$ is
(a) 0 (b) 1 (c) –1 (d) $1 + i$

14. The domain of the function $f(x) = \log(3x^2 - 4x + 7)$ is
(a) set of real numbers $-\{3, 2, 7\}$
(b) set of real numbers

(c) set of positive real numbers
(d) set of non-negative real numbers

15. $\int_0^{\pi/4} \tan^2 x \sec^4 x dx =$

(a) 1/3 (b) 4/15 (c) 1 (d) 8/15

16. Out of 4 symbols, 5 letters and 3 numbers, how many strings can be formed each containing 3 letters, 2 symbols and beginning with a number?
(a) 180 (b) 2160 (c) 14400 (d) 21600

17. There is a set P of ordered pairs in which each pair has a vowel as first element and a consonant as second element. It is given that $M = 4^{10}$. How many elements will be there in power set of P?
(a) $32(M^5)$ (b) $16(M^5)$ (c) $32(M^4)$ (d) $16(M^4)$

18. For 'C' is an arbitrary constant,

$$\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx =$$

(a) $3x - 2 + C$

(b) $\frac{1}{6}\log(x-1)-\frac{1}{3}\log(x+2)+\frac{1}{2}\log(x-3)+C$

(c) $-\frac{1}{6}\log(x-1)-\frac{1}{3}\log(x+2)+\frac{1}{2}\log(x-3)+C$

(d) $\frac{1}{6}\log(x-1)+\frac{1}{3}\log(x+2)+\frac{1}{2}\log(x-3)+C$

19. The solution of $\frac{dy}{dx} = (y+4x+1)^2$ when $y(0) = 1$, is given by
(a) $y = 2\tan(2x + \pi/4) - 4x - 1$
(b) $y = 2\cot(2x + \pi/8) - 4x - 1$
(c) $y = 2\tan(x + \pi/4) - 4x - 1$
(d) $y = 2\cot(2x + \pi/4) - 4x - 1$

20. A set $P = \{7, 21, 28, 42, 14, 0\}$ is given. A relation R holds on elements a and b of set P if either a is a factor of b or b is a factor of a. The relation R on the set P is
(a) NOT reflexive (b) NOT symmetric
(c) NOT transitive (d) antisymmetric

21. For 'C' is an arbitrary constant, the solution of the differential equation $\frac{dy}{dx} + 2 = e^{2x+y}$ is given by
(a) $(x + C)e^{(x + 2y)} + 1 = 0$
(b) $(2x + C)e^{(2x + y)} + 1 = 0$
(c) $(x + C)e^{(2x + y)} + 1 = 0$
(d) $(2x + y + C)e^{(2x + y)} + 1 = 0$

22. An eagle is sitting at a distance of 100 metres from base of a building. It can see a sparrow sitting at the top of the building at an angle of elevation of 30 degrees. It moves towards the building and sits at a point such that now it can see the same sparrow at an angle of elevation of 45 degrees. At what distance from the base of building is the eagle sitting now?
(a) 57.735 metres (b) 70.711 metres
(c) 100 metres (d) 173. 205 metres

23. We are given that p and q are two complex numbers such that $|p + q| = |p| + |q|$. The value of arg (p) – arg(q) is
(a) 0 (b) $\pi/2$ (c) $\pi/3$ (d) $3\pi/4$

24. What will be the area of the triangle formed by the lines joining the vertex of the parabola $y^2 = 28x$ to the ends of its latus rectum?
(a) 64 sq.units (b) 51 sq.units
(c) 98 sq.units (d) 72 sq. units

25. It is given that $f(x) = \frac{-3x+(-1)^x}{4x-(-1)^x}$. The limit value of the function $f(x)$ as x tends to infinity is
(a) 3/4 (b) –3/4 (c) 1/4 (d) –1/4

26. Which of the following can be the equal expression for $\cos(3T) + i\sin(3T)$ using the De Moivre's theorem and binomial theorem concepts?
(a) $\cos^3T + 3i\cos^2T\sin T - 3\cos T\sin^2T + i\sin^3T$
(b) $\cos^3T + 3i\cos^2T\sin T + 3\cos T\sin^2T - i\sin^3T$
(c) $\cos^3T + 3i\cos^2T\sin T - 3\cos T\sin^2T - i\sin^3T$
(d) $\cos^3T + 3i\cos^2T\sin T + 3\cos T\sin^2T + i\sin^3T$

27. A plane passes through the points (2, 1, –1) and (–1, 3, 4). Also, this plane is perpendicular to the plane $x - 2y + 4z = 10$. The equation of the plane is
(a) $7x + 9y + 6z = 23$ (b) $x + y + z = 2$
(c) $7x + y + z = 0$ (d) $18x + 17y + 4z = 49$

28. A parabola $y^2 = 32x$ is drawn. From its focus, a line of slope 1 is drawn. The equation of the line is
(a) $y = x + 8$ (b) $y = x - 4$
(c) $y = x$ (d) $y = x - 8$

29. A 3×3 square matrix $M = [a_{ij}]$ is taken, where $i = 0, 1, 2$ and $j = 0, 1, 2$. In matrix M, value of each element is determined as $a_{ij} = i \times j$. What will be the value of largest minor of the matrix M?
(a) –1 (b) 0 (c) 1 (d) 4

30. What will be the sum of coefficients of even powers of x in the expansion of $(1 + 3x)^{10}$?
(a) 524144 (b) 524414
(c) 524800 (d) 524288

31. What will be the argument of the complex number $z = \frac{i-1}{e^{i\pi/3}}$?
(a) $\pi/4$ (b) $\pi/3$ (c) $5\pi/12$ (d) $3\pi/8$

32. For which of the following values of T, the expression $(1 + i\cos T)/(1 - 2i\cos T)$ will give a real number?
(a) 0 (b) $\pi/2$ (c) $\pi/3$ (d) $3\pi/4$

33. For 'C' is an arbitrary constant, the solution of the differential equation $3(xy)^2dx + 2x^3ydy = 0$ is given by
(a) $x^2y^2 + C = 0$ (b) $x^3y^3 + C = 0$
(c) $x^3y^2 + C = 0$ (d) $x^2y^3 + C = 0$

34. What is the number of different values of the products that can be done using two or more of the numbers 3, 4, 5 and 7? (A number can be used at most once)
(a) 11 (b) 12 (c) 15 (d) 24

35. A function $f(x) = x^2\tan x + e^{2x}$ is given. The value of second order derivative of $f(x)$ at $x = 0$ is
(a) 1 (b) 2 (c) 4 (d) 0

36. The sum value of the infinite series
$1+\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\frac{1}{625}+....\infty$ is
(a) $\frac{4}{5}$ (b) $\frac{5}{4}$ (c) $\frac{3}{2}$ (d) indefinite

37. A set P is taken such that $P = \{2/3^p | p$ is a natural number$\}$. What will be the sum of elements of set P?
(a) 1 (b) 2/3 (c) 5/3 (d) 2

38. Two fair coins are thrown together. Only if heads appear on both coins, a fair dice is thrown on which natural numbers from 1 to 6 are written on the faces. What is the probability that face on which 2 is written will appear on the dice?
(a) 1/4 (b) 1/6 (c) 1/12 (d) 1/24

39. Which of the following functions is neither even nor odd?
(a) $f(x) = 5x + \sin(4x)$ (b) $f(x) = 4x^3 + 7\tan x$
(c) $f(x) = 7x^4 + 8x^2 - 6x$ (d) $f(x) = 5x^2 + \cos(6x)$

40. Suppose A, B and C are three sets, each with three elements. The number of subsets of the set $A \times B \times C$ that have at least 2 elements is
(a) $(2^{27}) - 28$ (b) $(2^{27}) - 55$
(c) 27 (d) $(27^2) - 3$

41. Nine non-zero integers a, b, c, d, e, f, g, h and i are taken such that they form a geometric progression in the same order as mentioned. The common ratio of the geometric progression is an integer. Which of the following relation will always hold?
(a) $abce > 0$ (b) $bcef > 0$
(c) $ahbd > 0$ (d) $bdgh > 0$

42. A system of linear equations in two variables, p and q, is given as $(n + 1)p + (n + 2)q = 8$, $p - (n + 1)q + (n + 2) = 0$, $p + q = 3$. Which of the following would be one of the values of n for which the given system of linear equations is consistent?
(a) 0 (b) 1 (c) 3 (d) 7

43. The mean of a group of data is given as 27. Data is 39, 21, 18, X, Y, 24. What could be the values of X and Y, if both X and Y are prime?
(a) $X = 23$, $Y = 29$ (b) $X = 29$, $Y = 31$
(c) $X = 26$, $Y = 34$ (d) $X = 12$, $Y = 48$

44. A set T is given as $T = \{x \in R | (x-3) = (x+4)^2, x < 2\}$. Which of these would be TRUE about the set T?
(a) T is an empty set (b) T is a singleton set
(c) $x = 3$ is a member of T
(d) $x = -4$ is a member of T

45. The probability that Madhu will pass in her terminal exam is 1/3. The probability that Meena will fail in same exam is 3/4. What is the probability that exactly one of them passes in this exam?
(a) 5/12 (b) 1/4 (c) 1/6 (d) 11/24

46. What will be the distance of point (2, 4, –1) from the straight line given by $(x + 5) = (y + 3)/4 = (6 - z)/9$?
(a) 5 units (b) 6 units (c) 7 units (d) 9 units

47. In which of the following domains $f(x)$ is continuous, where $f(x) = \frac{|x+2|}{\tan^{-1}(x+2)}$?
(Note: R denotes set of real numbers)
(a) R (b) $R - \{-2, 2\}$
(c) $R - \{-2\}$ (d) $\{-2, 2\}$

48. A function is given by the equations in parametric form as $x = 2\cos t - \cos(2t)$, $y = 2\sin t - \sin(2t)$. The value of $\frac{dy}{dx}$ at $t = 90°$ is
(a) 0 (b) –1 (c) 1 (d) 2

49. To the curve $x = \sin t + 2t$, $y = 4t$, a normal is drawn at $t = 0$. Where would this normal intersect with line $y = x + 3$?
(a) (6, 9) (b) (9, 12) (c) (0, 3) (d) (4, 7)

50. What will be the area enclosed by the ellipse given by $\frac{4x^2}{81}+\frac{9y^2}{49}=1$?
(a) 16.49 sq. units (b) 24.72 sq. units
(c) 32.97 sq.units (d) 48 sq. units

51. The sum of the series :
$\sin p + x\sin(p+q)+\frac{x^2}{2!}\sin(p+2q)+...\infty$ is
(a) 0 (b) $e^{x\cos q}[\sin(p + x\sin q)]$
(c) 1 (d) $e^{x\cos q}[\cos(p + x\sin q)]$

52. If $-\pi/2 < p < \pi/2$, then the values of 'p' for which $2\sin^2 p - 7\sin p + 3 > 0$, lie in
(a) $(0, \pi/2)$ (b) $(-\pi/2, -\pi/6)$
(c) $(-\pi/2, \pi/6)$ (d) $(-\pi/2, 0)$

53. Two matrices, A and B are both 5×5 square matrices such that $A = kB$, where k is a non-zero constant. If $|B| = z$, then what will be the value of $|4(A + B)|$?
(a) $5z$ (b) $(20k + 20)z$
(c) $3125z$ (d) $(4k + 4)^5 z$

54. If $S = \sin^2 p - \frac{1}{2}\sin(2p)(\sin^2 p)+\frac{1}{3}\sin(3p)(\sin^3 p) - \frac{1}{4}\sin(4p)(\sin^4 p) + ... \infty$ then $\tan S =$
(a) $\frac{\cos^2 p}{1+\cos p\cdot\sin p}$ (b) $\frac{\sin^2 p}{1+\cos p\cdot\sin p}$
(c) $\frac{\sin^2 p}{1+\cos 2p}$ (d) 0

55. If $f(x)=\sqrt{[(\sin 2x+2)(\cos 2x+2)]}$ for x is a real number, then which of the following values lies in the domain of $f(x)$ but NOT in the range of $f(x)$?
(a) $\sqrt{6}$ (b) $\sqrt{2}$
(c) 1 (d) $(1/\sqrt{2})+2$

56. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex of the triangle is at the vertex of the parabola. The length of the side of the triangle is
(a) $8a$ (b) $16a$ (c) $8a\sqrt{3}$ (d) $16a\sqrt{3}$

57. The direction ratios of a normal to a plane are (3, 12, 4). Through which of the following points the plane passes if origin is at a distance of 2 units from this plane?
(a) (2, 2, 1) (b) (2, 1, 2)
(c) (2, 1, 1) (d) (3, –1, 2)

58. Two lines in three-dimensional system are given as $(x - 1)/2 = (y - 2)/3 = (z - 3)/4$ and $(x - 4)/5 = (y - 1)/2 = z$. Their point of intersection is
(a) (1, 1, 1) (b) (1, 2, 3)
(c) (–1, –1, –1) (d) (4, 1, 0)

59. $[(\cos 3t + i\sin 3t)^4(\cos 4t - i\sin 4t)^5(\cos 5t + i\sin 5t)^4] \div(\cos 4t + i\sin 4t)^3 =$
(a) 0 (b) 1/2 (c) 1 (d) $8\sqrt{2}$

60. What will be the coordinates of foot of perpendicular from the point (1, 1.5, 2) to the plane $2x - 2y + 4z + 5 = 0$?
(a) (1, 1/2, 1) (b) (0, 5/2, 0)
(c) (0, 0, –5/4) (d) (6, 1 –15/4)

SOLUTIONS

1. (d) : Given that $P(n) \propto n$ *i.e.* $P(n)=\frac{n}{k}$, where k is constant of proportionality.

Now, $\sum_{i=1}^{6} P(x_i)=1 \Rightarrow \frac{1}{k}+\frac{2}{k}+\frac{3}{k}+\frac{4}{k}+\frac{5}{k}+\frac{6}{k}=1$

Favourable outcomes are {(3, 3), (3, 6), (6, 3), (6, 6)}

∴ Required probability

$=\frac{3}{21}\times\frac{3}{21}+\frac{3}{21}\times\frac{6}{21}+\frac{6}{21}\times\frac{3}{21}+\frac{6}{21}\times\frac{6}{21}=\frac{81}{441}=\frac{9}{49}$

2. (c) : $\arg(z)=\tan^{-1}\left(\frac{\sin p}{1-\cos p}\right)$

$=\tan^{-1}\left(\frac{2\sin p/2\cos p/2}{1-(1-2\sin^2 p/2)}\right)$

$=\tan^{-1}\left(\frac{\cos p/2}{\sin p/2}\right)=\tan^{-1}\left(\cot\frac{p}{2}\right)$

$=\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\frac{p}{2}\right)\right)=\frac{\pi}{2}-\frac{p}{2}$

3. (b) : We have, $f'(x) = \sec^4 x + 5$

$\Rightarrow \int f'(x)dx=\int\sec^4 x dx+\int 5dx$

$\Rightarrow f(x)=\tan x+\frac{1}{3}\tan^3 x+5x+c$

Now, $f(0)=\tan(0)+\frac{1}{3}\tan^3(0)+5\times0+c=0 \Rightarrow c=0$

4. (d) : Given plane $3x + 4y - 12z + 13 = 0$ is equidistant from the points $A(1, 1, k)$ and $B(-3, 0, 1)$

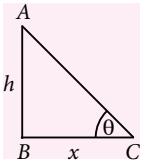
$\therefore \quad \frac{|3(-3)+4(0)-12(1)+13|}{\sqrt{(3)^2+(4)^2+(-12)^2}} = \frac{|3(1)+4(1)-12(k)+13|}{\sqrt{(3)^2+(4)^2+(-12)^2}}$

$\Rightarrow |-9 - 12 + 13| = |3 + 4 - 12k + 13|$

$\Rightarrow 8 = \pm (20 - 12k) \Rightarrow k = 1 \text{ or } \frac{7}{3}$

So, the required equation is $3x^2 - 10x + 7 = 0$

5. (c) : Let height of tower $AB = h$
And BC be the shadow of tower.
According to question, $h^2 = 3x$



In ΔABC, $\tan\theta = \frac{h}{x}$

$\tan\theta = \frac{\sqrt{3x^2}}{x} = \sqrt{3} = \tan 60° \Rightarrow \theta = 60°$

The angle of depression of sun at this moment is 60°.

6. (c) : We have,

$f(x) = \int_2^5 2[x]dx = 2\left[\int_2^3 2dx + \int_3^4 3dx + \int_4^5 4dx\right]$

$= 2\{[2x]_2^3 + [3x]_3^4 + [4x]_4^5\} = 2[2 + 3 + 4] = 18$

7. (c) : We have, $f(x) = [x]^2 + x^2$
Given, $f(x) > 25$
$\therefore \quad [x]^2 + x^2 > 25$
$\Rightarrow (x - \{x\})^2 + x^2 > 25$
$[\because x = [x] + \{x\} \text{ where } 0 \le \{x\} < 1]$
$\Rightarrow x^2 + \{x\}^2 + x^2 - 2x\{x\} > 25$
$\Rightarrow 2x^2 - 2x\{x\} + \{x\}^2 > 25$
$\Rightarrow 2x(x - \{x\}) + \{x\}^2 > 25 \Rightarrow 2x[x] + \{x\}^2 > 25$
$\Rightarrow 2x[x] \ge 24 \quad [\because 0 \le \{x\}^2 < 1]$
$\Rightarrow x[x] \ge 12$, which is possible for $x \ge 4$ or $x \le -4$
$\therefore$ Value of x is the member of the set
$\{x \mid x \le -4 \text{ or } x \ge 4\}$

8. (b) : We have 32, 16, 8, 4 .... are in G.P.

$a = 32, \ r = \frac{1}{2}$

Then, 12^{th} term of the sequence $= ar^{12-1}$

$= 32 \times \frac{1}{2^{11}} = 2^5 \times 2^{-11} = 2^{-6} = 0.015625$

Hence, six digits will be there after decimal point in 12^{th} term of the sequence.

9. (b) : Total black cards = 26
Remaining no. of cards = 52 – 13 = 39

$\therefore$ Required probability $= \frac{26}{39} = \frac{2}{3}$

10. (d) **11. (a)**

12. (a) : We have, foci = $(0, \pm be) = 0, \pm 10)$
and latus rectum = $30 = 2a^2/b \Rightarrow a^2 = 15b$

i.e., $e = \sqrt{1 + \frac{15b}{b^2}} \Rightarrow e^2 = 1 + \frac{15e}{10}$

$\Rightarrow 2e^2 - 3e - 2 = 0 \Rightarrow (e - 2)(2e + 1) = 0$

$\Rightarrow e = 2 \text{ or } -\frac{1}{2}$

If $e = 2$, then $b = \frac{10}{2} = 5 \Rightarrow a^2 = 15 \times 5 = 75$

Hence, equation of hyperbola is $\frac{x^2}{75} - \frac{y^2}{25} = -1$

$\Rightarrow x^2 - 3y^2 = -75 \text{ or } 3y^2 - x^2 = 75$

13. (c) : We have, $z_r = \cos\left(\frac{\pi}{2^r}\right) + i\sin\left(\frac{\pi}{2^r}\right)$

$\therefore \quad z_1 \cdot z_2 \cdot z_3 \cdot ..... \infty$

$= \left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right]\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$

$\times \left[\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right] \times .....\infty$

$= e^{i\pi/2} \cdot e^{i\pi/4} \cdot e^{i\pi/8} ..... \infty$

$= e^{i\pi\left[\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ...\infty\right]} = e^{i\pi\left[\frac{1/2}{1-1/2}\right]} = e^{i\pi}$

$= \cos\pi + i\sin\pi = -1 \quad (\because \sin\pi = 0)$

14. (b)

15. (d) : Let $I = \int_0^{\pi/4} \tan^2 x \sec^4 x dx$

$= \int_0^{\pi/4} \tan^2 x \cdot \sec^2 x \cdot \sec^2 x dx$

$= \int_0^{\pi/4} \tan^2 x(1 + \tan^2 x)\sec^2 x dx$

Put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$
When $x = 0$, $t = 0$ and $x = \pi/4$, $t = 1$

$\therefore \quad I = \int_0^1 t^2(1 + t^2)dt = \left[\frac{t^3}{3} + \frac{t^5}{5}\right]_0^1 = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

16. (d) : Required number of ways $= {}^3C_1 \times {}^5C_3 \times {}^4C_2 \times 5!$
$= 180 \times 120 = 21600$

17. (a) : Total number of vowels in English alphabets = 5
Total number of consonants in English alphabets = 21
Since, set P has ordered pairs in which each pair has a vowel as first element and a consonant as second element.

$\therefore$ Total number of element in P = 21 + 21 + 21 + 21+ 21 = 105

Now, number of elements in power set of P

$= 2^{105} = 2^{2\times 50+5} = 2^5 \cdot 4^{50} = 2^5 \cdot (4^{10})^5$

$= 2^5 \cdot (M)^5 = 32(M^5)$

18. (c) : Let $I = \int \frac{(2x-1)}{(x-1)(x+2)(x-3)}dx$ ...(i)

We first decompose the integrand into partial fractions.

$\therefore \quad \frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x-3}$

$\Rightarrow 2x - 1 = A(x + 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x + 2)$

At $x=-2,\ -5=15B \Rightarrow B=\frac{-1}{3}$

At $x=1,\ 1=-6A \Rightarrow A=\frac{-1}{6}$

At $x=3,\ 5=10C \Rightarrow C=\frac{1}{2}$

$\therefore \quad I=\int\left[\frac{-1}{6(x-1)}-\frac{1}{3(x+2)}+\frac{1}{2(x-3)}\right]dx$

$=\frac{-1}{6}\log(x-1)-\frac{1}{3}\log(x+2)+\frac{1}{2}\log(x-3)+C$

19. (a) : The given differential equation is,

$\frac{dy}{dx}=(y+4x+1)^2$ ...(i)

Put $(y + 4x + 1) = t \Rightarrow \frac{dy}{dx}+4=\frac{dt}{dx}$

$\therefore$ (i) becomes,

$\frac{dt}{dx}-4=t^2 \Rightarrow \frac{dt}{dx}=t^2+2^2 \Rightarrow \int\frac{dt}{t^2+2^2}=\int dx$

$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)=x+C$

$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{y+4x+1}{2}\right)=x+C$

Now, at $x = 0$, $y = 1$

$\therefore \quad \frac{1}{2}\tan^{-1}(1)=C \Rightarrow C=\frac{\pi}{8}$

So, the required solution is,

$\frac{1}{2}\tan^{-1}\left(\frac{y+4x+1}{2}\right)=x+\frac{\pi}{8}$

$\Rightarrow \quad y+4x+1=2\tan\left(2x+\frac{\pi}{4}\right)$

$\Rightarrow \quad y=2\tan\left(2x+\frac{\pi}{4}\right)-4x-1$

20. (c) : We have,

R = {(7, 7), (21, 21), (28, 28), (42, 42), (14, 14), (7, 21), (7, 28), (7, 42), (7, 14), (14, 7), (21, 42), (14, 28), (21, 7), (28, 7), (42, 7), (42, 21), (28, 14)}

Now, $(28, 7) \in R$, $(7, 42) \in R$ but $(28, 42) \notin R$

$\therefore$ R is not transitive.

21. (c) : The given differential equation is,

$\frac{dy}{dx}+2=e^{2x+y}$ ...(i)

Put $e^{2x+y} = t$

$\Rightarrow \quad e^{2x+y}\left(2+\frac{dy}{dx}\right)=\frac{dt}{dx} \Rightarrow 2+\frac{dy}{dx}=\frac{1}{t}\frac{dt}{dx}$

$\therefore$ (i) becomes, $\frac{1}{t}\frac{dt}{dx}=t \Rightarrow \int\frac{dt}{t^2}=\int dx$

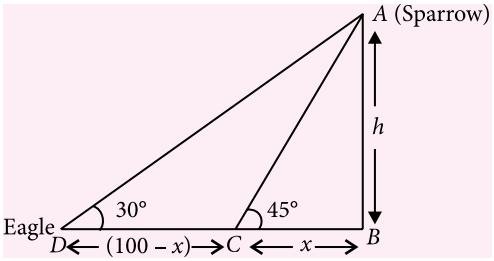
$\Rightarrow \quad -\frac{1}{t}=x+C \Rightarrow -1=(e^{2x+y})(x+C)$

$\Rightarrow \quad e^{2x+y}(x+C)+1=0$

22. (a) : Let h be the height of the building

In ΔABC, we have

$\tan 45^\circ=\frac{h}{x} \Rightarrow h = x$ ...(i)



In ΔABD, we have

$\tan 30^\circ=\frac{h}{100} \Rightarrow h=\frac{100}{\sqrt{3}}$ m

$\therefore$ From (i), we have,

$x=\frac{100}{\sqrt{3}}\text{ m}=57.735\text{ m}$

23. (a) Let $p = x_1 + iy_1$, $q = x_2 + iy_2$

$\therefore \quad |p+q|=\sqrt{(x_1+x_2)^2+(y_1+y_2)^2}$

$\Rightarrow \quad |p|+|q|=\sqrt{(x_1+x_2)^2+(y_1+y_2)^2}$ $(\because |p + q| = |p| + |q|)$

$\Rightarrow \quad \sqrt{x_1^2+y_1^2}+\sqrt{x_2^2+y_2^2}=\sqrt{(x_1+x_2)^2+(y_1+y_2)^2}$

Squaring both sides, we get

$(x_1 + x_2)^2 + (y_1 + y_2)^2$

$=(x_1^2+y_1^2)+(x_2^2+y_2^2)+2\sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}$

$\Rightarrow \quad 2x_1x_2+2y_1y_2=2\sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}$

Squaring both sides, we get

$4x_1^2x_2^2 + 4y_1^2y_2^2 + 8x_1x_2y_1y_2 = 4x_1^2x_2^2 + 4y_1^2y_2^2 + 4x_1^2y_2^2 + 4x_2^2y_1^2$

$\Rightarrow 4[x_1y_2 - y_1x_2]^2 = 0$

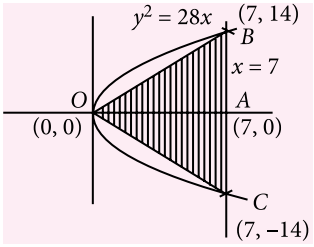
$\Rightarrow x_1y_2 = y_1x_2 \Rightarrow \frac{y_1}{x_1} = \frac{y_2}{x_2}$

$\Rightarrow \tan^{-1}\left(\frac{y_1}{x_1}\right) - \tan^{-1}\left(\frac{y_2}{x_2}\right) = 0$

$\Rightarrow \text{Arg}(p) - \text{Arg}(q) = 0$

24. (c) : The given equation of parabola is,

$y^2 = 28x = 4 \times 7 \times x$



Equation of latus rectum is $x - 7 = 0$

So, required area = 2 × Area of ΔOAB

$= 2 \times \frac{1}{2} \times \text{Base} \times \text{Height} = 7 \times 14 = 98$ sq. units

25. (b)

26. (c) : According to De Moivre's theorem,

$\cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^n \ \forall\ n \in I$

Put $n = 3$, $\theta = T$, we get

$\cos 3T + i\sin 3T = (\cos T + i\sin T)^3$

$= {}^3C_0(\cos T)^3 + {}^3C_1(\cos T)^2(i\sin T)^1 + {}^3C_2(\cos T)^1(i\sin T)^2 + {}^3C_3(i\sin T)^3$

$= \cos^3 T + 3i\cos^2 T \sin T - 3\cos T\sin^2 T - i\sin^3 T$

27. (d) : Let the required equation of plane be

$ax + by + cz + d = 0$ ...(i)

Now, plane (i) passes through (2, 1, –1) and (–1, 3, 4)

$\therefore\ 2a + b - c + d = 0$ ...(ii)

$-a + 3b + 4c + d = 0$ ...(iii)

Solving (ii) and (iii), we get

$3a - 2b - 5c = 0$ ...(iv)

Also, plane (i) is perpendicular to the plane

$x - 2y + 4z = 10$

$\therefore\ a - 2b + 4c = 0$ ...(v)

From (iv) and (v), we have

$\frac{a}{-8-10} = \frac{b}{-5-12} = \frac{c}{-6+2} \Rightarrow a = -18,\ b = -17,\ c = -4$

Now, from (ii), we have

$d = 36 + 17 - 4 = 49$

So, required equation of plane is,

$-18x - 17y - 4z + 49 = 0$

$\Rightarrow\ 18x + 17y + 4z = 49$

28. (d) : The given parabola is $y^2 = 32x$ ...(i)

Focus = (8, 0); Slope = 1

$\therefore$ Equation of line passing through (8, 0) and having slope 1 is, $y - 0 = 1(x - 8) \Rightarrow y = x - 8$

29*. (b) : The question is incorrect.

Since i, j can't be 0.

It should be $i, j = 1, 2, 3$

$\therefore\ a_{ij} = i \times j$, where $i, j = 1, 2, 3$

$\therefore\ M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

30. (c) : Required sum $= {}^{10}C_0 + {}^{10}C_2 \cdot 3^2 + {}^{10}C_4 \cdot 3^4 + {}^{10}C_6 \cdot 3^6 + {}^{10}C_8 \cdot 3^8 + {}^{10}C_{10} \cdot 3^{10}$

$= {}^{10}C_0(1 + 3^{10}) + {}^{10}C_2(3^2 + 3^8) + {}^{10}C_4(3^4 + 3^6)$

$= 59050 + 45 \times 6570 + 210 \times 810 = 524800$

31. (c) : We have, $z = \frac{i-1}{e^{i\pi/3}} = \frac{i-1}{\cos(\pi/3) + i\sin(\pi/3)}$

$\Rightarrow z = 2\left[\frac{(i-1)}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}\right] = 2\left[\frac{i-1+\sqrt{3}+i\sqrt{3}}{1+3}\right]$

$= \frac{1}{2}[(\sqrt{3}-1) + i(\sqrt{3}+1)]$

$\therefore\ \arg(z) = \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) = \tan^{-1}\left(\frac{1+1/\sqrt{3}}{1-1/\sqrt{3}}\right)$

$= \tan^{-1}(1) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$

32. (b) : Let $z = \frac{1 + i\cos T}{1 - 2i\cos T}$

$= \frac{1+i\cos T}{1-2i\cos T} \times \frac{1+2i\cos T}{1+2i\cos T}$

$= \frac{1 + i\cos T + 2i\cos T - 2\cos^2 T}{1 + 4\cos^2 T}$

$= \frac{-(2\cos^2 T - 1) + i3\cos T}{1 + 4\cos^2 T}$

$\because z$ is real $\therefore\ 3\cos T = 0 \Rightarrow \cos T = \cos\pi/2 \Rightarrow T = \pi/2$

33. (c) : The given differential equation is,

$3(xy)^2dx + 2x^3ydy = 0$

$\Rightarrow\ 2x^3ydy = -3x^2y^2dx$

$\Rightarrow \int\frac{dy}{y} = \frac{-3}{2}\int\frac{dx}{x} \Rightarrow \log y = \frac{-3}{2}\log x + \log C_1$

$\Rightarrow\ 2\log y + 3\log x = 2\log C_1$

$\Rightarrow\ y^2 \cdot x^3 - C_1^2 = 0$

$\Rightarrow\ y^2x^3 + C = 0$

CONCEPT MAP

ELLIPSE

Class XI Class XII

Definition
Locus of a point which moves in such a way that the sum of its distances from two fixed points is always constant.

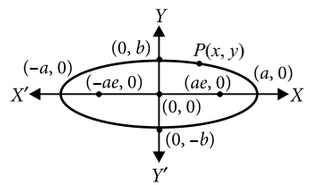
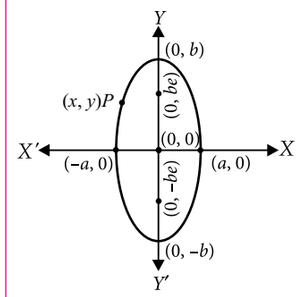
Tangent
Equation of tangent is given by

- At (x_1, y_1) : $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$
- At$(a\cos\theta, b\sin\theta)$: $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$
- In slope form : $y=mx\pm\sqrt{a^2m^2+b^2}$
- Equation of pair of tangent $S_1^2=S_{11}S$, where $S\equiv\frac{x^2}{a^2}+\frac{y^2}{b^2}-1, S_1\equiv\frac{x_1x}{a^2}+\frac{y_1y}{b^2}-1,\ S_{11}\equiv\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$
- Equation of chord of contact of tangents drawn from a point (x_1, y_1) to ellipse is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

Condition of tangency for a line to the ellipse

Ellipse	Equation of line	Condition
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$	$y=mx+c$	$c^2=a^2m^2+b^2$
	$x\cos\alpha+y\sin\alpha=p$	$p^2=a^2\cos^2\alpha+b^2\sin^2\alpha$
	$lx+my+n=0$	$n^2=a^2l^2+b^2m^2$

Standard Form
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a<b$
Graph		
Centre	(0, 0)	(0, 0)
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Length of major axis	$2a$	$2b$
Length of minor axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$x=\pm\left(\frac{a}{e}\right)$	$y=\pm\left(\frac{b}{e}\right)$
Eccentricity	$e=\sqrt{1-\frac{b^2}{a^2}}$	$e=\sqrt{1-\frac{a^2}{b^2}}$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

Normal
Equation of normal is given by

- At (x_1, y_1) : $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$
- At$(a\cos\theta, b\sin\theta)$: $ax\sec\theta-by\operatorname{cosec}\theta=a^2-b^2$
- Slope form : $y=mx-\frac{m(a^2-b^2)}{\sqrt{a^2+b^2m^2}}$

PLANE

DEFINITION
A plane is a surface such that line joining any two points totally lies in it.

EQUATION OF PLANE IN DIFFERENT FORMS

Normal Form
- **Vector form :** $\vec{r}\cdot\hat{n}=d$, where d is distance of plane from origin and $\hat{n}$ is unit vector normal to the plane.
- **Cartesian form :** $lx+my+nz=d$, where l, m, n are direction cosines of normal to plane.

One Point Form
- **Vector form :** $(\vec{r}-\vec{a})\cdot\vec{N}=0$
- **Cartesian form :** $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where a, b, c are dr's of normal to plane.

Equation of Plane Through Three non-collinear Points
- **Vector form :** $(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$, where $\vec{a},\vec{b}$ and $\vec{c}$ are position vectors of given points.
- **Cartesian form :** If (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3), be three points then $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$

Intercept Form
If intercepts on x, y, z axes are a, b, c, then $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Equation of Plane Passing Through Intersection of Two Planes
- **Vector form :** Equation of plane passing through the intersection of the plane $\vec{r}\cdot\vec{n}_1=d_1$ and $\vec{r}\cdot\vec{n}_2=d_2$ is given by $(\vec{r}\cdot\vec{n}_1-d_1)+\lambda(\vec{r}\cdot\vec{n}_2-d_2)=0$ *i.e.,* $\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$
- **Cartesian form :** The equation of plane passing through the intersection of $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is given by $(a_1x+b_1y+c_1z+d_1)+\mu(a_2x+b_2y+c_2z+d_2)=0$

Angle Between Two Planes
Vector form : For $\vec{r}\cdot\vec{n}_1=d_1$ and $\vec{r}\cdot\vec{n}_2=d_2$ then $\cos\theta=\pm\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

If $\vec{n}_1=a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and $\vec{n}_2=a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$
- Planes are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$ (Cartesian form) and $\vec{n}_1\cdot\vec{n}_2=0$ (Vector form)
- Planes are parallel if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ (Cartesian form) and $\vec{n}_1=\lambda\vec{n}_2$ (Vector form)

Distance of a Point from a Plane
Vector form : $\frac{|\vec{a}\cdot\vec{n}-d|}{|\vec{n}|}$

where $\vec{a}$ = position vector of the point and plane is $\vec{r}\cdot\vec{n}=d$

Cartesian form : Perpendicular distance of a point (x_1, y_1, z_1) from a plane $ax+by+cz+d=0$ is $p=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

Distance Between Parallel Planes
If $\vec{r}\cdot\vec{n}=d_1$ and $\vec{r}\cdot\vec{n}=d_2$ be two parallel planes then $d=\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$ where $\vec{n}=a\hat{i}+b\hat{j}+c\hat{k}$

Equation of Bisector of Two Planes
Vector form : $r\cdot\vec{n}_1=d_1$ and $\vec{r}\cdot\vec{n}_2=d_2$ then, $\frac{|\vec{r}\cdot\vec{n}_1-d_1|}{|\vec{n}_1|}=\frac{|\vec{r}\cdot\vec{n}_2-d_2|}{|\vec{n}_2|}$

Cartesian form : If $a_1x+b_1y+c_1y+d_1=0$ and $a_2x+b_2y+c_2y+d_2=0$ then $\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}=\pm\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}}$

Image of a Point in a Plane
$\frac{x'-x_1}{a}=\frac{y'-y_1}{b}=\frac{z'-z_1}{c}=\frac{-2|ax_1+by_1+cz_1+d|}{a^2+b^2+c^2}$

34. (a) : Possible combinations are (3, 4), (3, 5), (3, 7), (4, 5), (4, 7), (5, 7), (3, 4, 5), (3, 4, 7), (3, 5, 7), (4, 5, 7), (3, 4, 5, 7)

$\therefore$ Required number = 11

35. (c) : We have, $f(x) = x^2\tan x + e^{2x}$

Differentiating w.r.t. x, we get

$f'(x) = x^2 \cdot \sec^2 x + \tan x \cdot 2x + e^{2x} \cdot 2$

$= x^2\sec^2 x + 2x\tan x + 2e^{2x}$

Again differentiating w.r.t x, we get

$f''(x) = x^2 \cdot 2\sec x \cdot \sec x\tan x + \sec^2 x \cdot 2x + 2(x \cdot \sec^2 x + \tan x) + 2e^{2x} \cdot 2$

$= 2x^2\sec^2 x \cdot \tan x + 4x\sec^2 x + 2\tan x + 4e^{2x}$

$\therefore \quad f''(0) = 0 + 0 + 0 + 4e^0 = 4$

36. (b) : The given series is,

$$1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+.....\infty$$

which is a G.P. with first term $(a) = 1$

and common ratio $(r) = \frac{1}{5}$

$$\therefore \quad \text{Sum of infinite terms} = \frac{1}{1-\frac{1}{5}} = \frac{5}{4}$$

37. (a) : We have, $P = \left\{\frac{2}{3^p} \mid p \in N\right\}$

$$\therefore \quad P = \left\{\frac{2}{3}, \frac{2}{3^2}, \frac{2}{3^3} .....\text{up to } \infty\right\}$$

$$\therefore \quad \text{Sum of elements of } P = \frac{2}{3}+\frac{2}{3^2}+\frac{2}{3^3}+.....\infty$$

$$= \frac{2/3}{1-1/3} \quad (\text{sum of infinite G.P}) = 1$$

38. (d) : Required probability $= \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

39. (c): (a) $f(-x) = -5x + \sin(-4x) = -(5x + \sin 4x) = -f(x)$

$\therefore$ $f(x)$ is odd.

(b) $f(-x) = -4x^3 + 7\tan(-x) = -(4x^3 + 7\tan x) = -f(x)$

$\therefore$ $f(x)$ is odd.

(c) $f(-x) = 7x^4 + 8x^2 + 6x \neq f(x)$

$\therefore$ $f(x)$ is neither even nor odd.

(d) $f(-x) = 5x^2 + \cos(-6x) = 5x^2 + \cos 6x = f(x)$

$\therefore$ $f(x)$ is even function.

40. (a) : Number of elements in $A \times B \times C = 3 \times 3 \times 3 = 27$

$\therefore$ Number of subsets of the set $A \times B \times C = 2^{27}$

Number of subsets having 1 element = 27

Number of subsets having 0 element = 1

$\therefore$ Required number of subsets $= 2^{27} - (27 + 1)$

$= 2^{27} - 28$

41. (b) : We have, $a, b, c, d, e, f, g, h, i \in$ G.P.

Let r be the common ratio.

$\therefore$ The terms becomes,

$a, b = ar, c = ar^2, d = ar^3, e = ar^4, f = ar^5, g = ar^6, h = ar^7, i = ar^8$

Now, from the options consider, $bcef$

$= ar \times ar^2 \times ar^4 \times ar^5 = a^4r^{12}$, which is always positive.

42. (b) : Given, system of linear equations is

$(n + 1)p + (n + 2)q = 8$ ...(i)

$p - (n + 1)q = -(n + 2)$ ...(ii)

$p + q = 3$ ...(iii)

Given that system is consistent, so values of p and q given by (i) and (ii) will satisfy (iii).

Multiplying (ii) by $(n + 1)$ and then subtracting from (i), we get

$[(n + 2) + (n + 1)^2]q = 8 + (n + 2)(n + 1)$

$$\Rightarrow \quad q = \frac{8+(n+2)(n+1)}{(n+2)+(n+1)^2} = \frac{n^2+3n+10}{n^2+3n+3}$$

$$\therefore \quad p = -(n+2)+(n+1)\left(\frac{n^2+3n+10}{n^2+3n+3}\right) = \frac{-n^2+4n+4}{n^2+3n+3}$$

From (iii), $p + q = 3$

$$\Rightarrow \quad \frac{-n^2+4n+4}{n^2+3n+3} + \frac{n^2+3n+10}{n^2+3n+3} = 3$$

$\Rightarrow 7n + 14 = 3(n^2 + 3n + 3)$

$\Rightarrow 3n^2 + 9n + 9 - 7n - 14 = 0$

$\Rightarrow 3n^2 + 2n - 5 = 0$

$$\Rightarrow \quad (3n+5)(n-1) = 0 \quad \Rightarrow \quad n = \frac{-5}{3} \text{ or } n = 1$$

43. (b) : We have, mean $(\bar{x}) = 27$

$$\Rightarrow \quad \frac{39+21+18+X+Y+24}{6} = 27$$

$\Rightarrow 102 + X + Y = 162 \Rightarrow X + Y = 60$

$\because$ X, Y are primes

$\therefore$ $X = 29, Y = 31$

44. (a) : We have, $T = \{x \in R : (x - 3) = (x + 4)^2, x < 2\}$

Now, if $x = 1$, we have

$1 - 3 = (1 + 4)^2$ which is not true.

$\because$ Square of a number can't be negative.

$\therefore$ T is an empty set.

45. (a)	**46. (c)**	**47. (c)**	**48. (b)**
49. (None)	**50. (c)**	**51. (b)**	**52. (c)**
53. (d)	**54. (b)**	**55. (c)**	**56. (c)**
57. (b)	**58. (c)**	**59. (c)**	**60. (b)**

For complete solutions refer to MTG J&K CET Explorer.

SOLVED PAPER **2018**

Kerala PET

1. The value of $\frac{2(\cos 75^\circ + i\sin 75^\circ)}{0.2(\cos 30^\circ + i\sin 30^\circ)}$ is

(a) $\frac{5}{\sqrt{2}}(1+i)$ (b) $\frac{10}{\sqrt{2}}(1+i)$

(c) $\frac{10}{\sqrt{2}}(1-i)$ (d) $\frac{5}{\sqrt{2}}(1-i)$

(e) $\frac{1}{\sqrt{2}}(1+i)$

2. If the conjugate of a complex number z is $\frac{1}{i-1}$, then z is

(a) $\frac{1}{i-1}$ (b) $\frac{1}{i+1}$ (c) $\frac{-1}{i-1}$ (d) $\frac{-1}{i+1}$

(e) $\frac{1}{i}$

3. The value of $\left(i^{18} + \left(\frac{1}{i}\right)^{25}\right)^3$ is equal to

(a) $\frac{1+i}{2}$ (b) $2 + 2i$

(c) $\frac{1-i}{2}$ (d) $\sqrt{2} - \sqrt{2}i$

(e) $2 - 2i$

4. The modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ is

(a) 2 (b) $\sqrt{2}$ (c) 4 (d) 8

(e) 10

5. If $z = e^{i4\pi/3}$, then $(z^{192} + z^{194})^3$ is equal to

(a) -2 (b) -1 (c) $-i$ (d) $-2i$

(e) 0

6. If a and b are real numbers and $(a + ib)^{11} = 1 + 3i$, then $(b + ia)^{11}$ is equal to

(a) $i + 3$ (b) $1 + 3i$

(c) $1 - 3i$ (d) 0

(e) $-i - 3$

7. If $\alpha \neq \beta$, $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(a) $3x^2 - 19x - 3 = 0$

(b) $3x^2 + 19x - 3 = 0$

(c) $x^2 + 19x + 3 = 0$

(d) $3x^2 - 19x - 19 = 0$

(e) $3x^2 - 19x + 3 = 0$

8. The focus of the parabola $y^2 - 4y - x + 3 = 0$ is

(a) $\left(\frac{3}{4}, 2\right)$ (b) $\left(\frac{3}{4}, -2\right)$

(c) $\left(2, \frac{3}{4}\right)$ (d) $\left(\frac{-3}{4}, 2\right)$

(e) $\left(2, \frac{-3}{4}\right)$

9. If $f : R \to (0, \infty)$ is an increasing function and if, $\lim_{x\to 2018} \frac{f(3x)}{f(x)} = 1$, then $\lim_{x\to 2018} \frac{f(2x)}{f(x)}$ is equal to

(a) 2/3 (b) 3/2 (c) 2 (d) 3

(e) 1

10. If f is differentiable at $x = 1$, then

$\lim_{x\to 1} \frac{x^2 f(1) - f(x)}{x - 1}$ is

(a) $-f'(1)$ (b) $f(1) - f'(1)$

(c) $2f(1) - f'(1)$ (d) $2f(1) + f'(1)$

(e) $f(1) + f'(1)$

11. Eccentricity of the ellipse $4x^2 + y^2 - 8x + 4y - 8 = 0$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{8}$

(e) $\frac{\sqrt{3}}{16}$

12. The focus of the parabola $(y + 1)^2 = -8(x + 2)$ is

(a) $(-4, -1)$ (b) $(-1, -4)$

(c) $(1, 4)$ (d) $(4, 1)$

(e) $(-1, 4)$

13. Which of the following is the equation of a hyperbola?

(a) $x^2 - 4x + 16y + 17 = 0$

(b) $4x^2 + 4y^2 - 16x + 4y - 60 = 0$

(c) $x^2 + 2y^2 + 4x + 2y - 27 = 0$

(d) $x^2 - y^2 + 3x - 2y - 43 = 0$

(e) $x^2 + 4x + 6y - 2 = 0$

14. Let $f(x) = px^2 + qx + r$, where p, q, r are constants and $p \neq 0$. If $f(5) = -3f(2)$ and $f(-4) = 0$, then the other root of f is

(a) 3 (b) −7 (c) −2 (d) 2

(e) 6

15. Let $f : R \to R$ satisfy $f(x)\, f(y) = f(xy)$ for all real numbers x and y. If $f(2) = 4$, then $f\left(\frac{1}{2}\right) =$

(a) 0 (b) 1/4 (c) 1/2 (d) 1

(e) 2

16. Sum of last 30 coefficients in the binomial expansion of $(1 + x)^{59}$ is

(a) 2^{29} (b) 2^{59}

(c) 2^{58} (d) $2^{59} - 2^{29}$

(e) 2^{60}

17. $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4 =$

(a) $20\sqrt{6}$ (b) $30\sqrt{6}$

(c) $5\sqrt{10}$ (d) $40\sqrt{6}$

(e) $10\sqrt{6}$

18. Three players A, B and C play a game. The probability that A, B and C will finish the game are respectively $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. The probability that the game is finished is

(a) 1/8 (b) 1 (c) 1/4 (d) 3/4

(e) 1/2

19. If $z_1 = 2 - i$ and $z_2 = 1 + i$, then $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$ is

(a) 2 (b) $\sqrt{2}$ (c) 3 (d) $\sqrt{3}$

(e) 1

20. If $f(x) = \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$, then $\lim_{x \to \infty} f(x)$ is equal to

(a) 1 (b) 2 (c) 1/2 (d) 0

(e) ∞

21. The value of $\sin\frac{31}{3}\pi$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{-\sqrt{3}}{2}$ (d) $\frac{-1}{\sqrt{2}}$

(e) $\frac{1}{2}$

22. The sum of odd integers from 1 to 2001 is

(a) $(1121)^2$ (b) $(1101)^2$

(c) $(1001)^2$ (d) $(1021)^2$

(e) $(1011)^2$

23. If $y = \frac{\sin^2 x}{1 + \cot x} + \frac{\cos^2 x}{1 + \tan x}$, then $y'(x)$ is equal to

(a) $2\cos^2 x$ (b) $2\cos^3 x$

(c) $-\cos 2x$ (d) $\cos 2x$

(e) $3\cos x$

24. The foci of the hyperbola $16x^2 - 9y^2 - 64x + 18y - 90 = 0$ are

(a) $\left(\frac{24 \pm 5\sqrt{145}}{12}, 1\right)$ (b) $\left(\frac{21 \pm 5\sqrt{145}}{12}, 1\right)$

(c) $\left(1, \frac{24 \pm 5\sqrt{145}}{2}\right)$ (d) $\left(1, \frac{21 \pm 5\sqrt{145}}{2}\right)$

(e) $\left(\frac{21 \pm 5\sqrt{145}}{2}, -1\right)$

25. If the sum of the coefficients in the expansion of $(a^2x^2 - 2ax + 1)^{51}$ is zero, then a is equal to

(a) 0 (b) 1 (c) −1 (d) −2

(e) 2

26. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is

(a) 2.23 (b) 3.23

(c) 2.57 (d) 3.57

(e) 1.03

27. The mean and variance of a binomial distribution are 8 and 4 respectively. What is $P(X = 1)$?

(a) $\frac{1}{2^8}$ (b) $\frac{1}{2^{12}}$ (c) $\frac{1}{2^6}$ (d) $\frac{1}{2^4}$
(e) $\frac{1}{2^5}$

28. The number of diagonals of a polygon with 15 sides is
(a) 90 (b) 45 (c) 60 (c) 70
(e) 10

29. In a class, 40 % of students study maths and science and 60% of students study maths. What is the probability of a student studying science given the student is already studying maths?
(a) 1/3 (b) 1/6 (c) 2/3 (d) 1/5
(e) 1/4

30. The eccentricity of the conic $x^2 + 2y^2 - 2x + 3y + 2 = 0$ is
(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) $\sqrt{2}$
(e) 1

31. If the mean of a set of observations $x_1, x_2, ..., x_{10}$ is 20, then the mean of $x_1 + 4, x_2 + 8, x_3 + 12, ..., x_{10} + 40$ is
(a) 34 (b) 32 (c) 42 (d) 38
(e) 40

32. A letter is taken at random from the word "STATISTICS" and another letter is taken at random from the word "ASSISTANT". The probability that they are same letters is
(a) $\frac{1}{45}$ (b) $\frac{13}{90}$ (c) $\frac{19}{90}$ (d) $\frac{5}{18}$
(e) $\frac{9}{10}$

33. If $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $ax^2 + bx + c = 0$, then
(a) $a^2 - b^2 + 2ac = 0$ (b) $(a - c)^2 = b^2 + c^2$
(c) $a^2 + b^2 - 2ac = 0$ (d) $a^2 + b^2 + 2ac = 0$
(e) $a + b + c = 0$

34. If the sides of a triangle are 4, 5 and 6 cms. Then the area of the triangle is ..... sq. cms.
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}\sqrt{7}$ (c) $\frac{4}{15}$ (d) $\frac{4}{15}\sqrt{7}$
(e) $\frac{15}{4}\sqrt{7}$

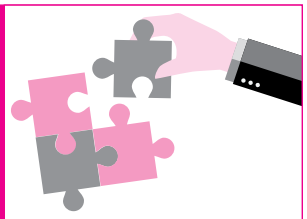
35. In a group of 6 boys and 4 girls, a team consisting of four children is formed such that the team has atleast one boy. The number of ways of forming a team like this is
(a) 159 (b) 209 (c) 200 (d) 240
(e) 212

36. A password is set with 3 distinct letters from the word LOGARITHMS. How many such passwords can be formed?
(a) 90 (b) 720 (c) 80 (d) 72
(e) 120

37. If 5^{97} is divided by 52, the remainder obtained is
(a) 3 (b) 5 (c) 4 (d) 0 (e) 1

38. A quadratic equation $ax^2 + bx + c = 0$, with distinct coefficients is formed. If a, b, c are chosen from the numbers 2, 3, 5, then the probability that the equation has real roots is
(a) 1/3 (b) 2/5 (c) 1/4 (d) 1/5
(e) 2/3

PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is multiple of its numbers. For example, if that value is 3 for a two-box cluster, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

11+	2÷		20×	6×	
	3−			3÷	
240×		6×			
		6×	7+	30×	
6×					9+
8+			2÷		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

39. $\lim_{x\to\infty} \frac{3x^2+2x^2-7x+9}{4x^3+9x-2}$ is equal to

(a) 2/9 (b) 1/2 (c) −9/2 (d) 3/4
(e) 9/2

40. The minimum value of $f(x) = \max\{x, 1 + x, 2 - x\}$ is

(a) 1/2 (b) 3/2 (c) 1 (d) 0
(e) 2

41. The equations of the asymptotes of the hyperbola $xy + 3x - 2y - 10 = 0$ are

(a) $x = -2, y = -3$ (b) $x = 2, y = -3$
(c) $x = 2, y = 3$ (d) $x = 4, y = 3$
(e) $x = 3, y = 4$

42. If $f(x) = x^6 + 6^x$, then $f'(x)$ is equal to

(a) $12x$ (b) $x + 4$
(c) $6x^5 + 6^x \log(6)$ (d) $6x^5 + x6^{x-1}$
(e) x^6

43. The standard deviation of the data 6, 5, 9, 13, 12, 8, 10, is

(a) $\frac{\sqrt{52}}{7}$ (b) $\frac{52}{7}$ (c) $\frac{\sqrt{53}}{7}$ (d) $\frac{53}{7}$
(e) 6

44. $\lim_{x\to 0} \frac{1-\cos mx}{1-\cos nx} =$

(a) $\frac{m^2}{n^2}$ (b) $\frac{n^2}{m^2}$ (c) ∞ (d) $-\infty$
(e) 0

45. $\lim_{x\to 0} \frac{\sqrt{1+2x}-1}{x} =$

(a) 0 (b) −1 (c) 1/2 (d) 1
(e) −1/2

46. Let f and g be differentiable functions such that $f(3) = 5, g(3) = 7, f'(3) = 13, g'(3) = 6, f'(7) = 2$ and $g'(7) = 0$. If $h(x) = (fog)(x)$, then $h'(3) =$

(a) 14 (b) 12 (c) 16 (d) 0
(e) 10

47. $\frac{\sqrt{3}}{\sin(20°)} - \frac{1}{\cos(20°)} =$

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) 4
(e) 0

48. A poisson variate X satisfies $P(X = 1) = P(X = 2)$. $P(X = 6)$ is equal to

(a) $\frac{4}{45}e^{-2}$ (b) $\frac{1}{45}e^{-1}$
(c) $\frac{1}{9}e^{-2}$ (d) $\frac{1}{4}e^{-2}$
(e) $\frac{1}{45}e^{-2}$

49. Let a and b be 2 consecutive integers selected from the first 20 natural numbers. The probability that $\sqrt{a^2+b^2+a^2b^2}$ is an odd positive integer is

(a) $\frac{9}{19}$ (b) $\frac{10}{19}$ (c) $\frac{13}{19}$ (d) 1
(e) 0

50. An ellipse of eccentricity $\frac{2\sqrt{2}}{3}$ is inscribed in a circle. A point is chosen inside the circle at random. The probability that the point lies outside the ellipse is

(a) 1/3 (b) 2/3 (c) 1/9 (d) 2/9
(e) 1/27

51. If the vectors $4\hat{i}+11\hat{j}+m\hat{k}$, $7\hat{i}+2\hat{j}+6\hat{k}$ and $\hat{i}+5\hat{j}+4\hat{k}$ are coplanar, then m is equal to

(a) 38 (b) 0 (c) 10 (d) −10
(e) 25

52. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+3\hat{j}+5\hat{k}$ and $\vec{c}=7\hat{i}+9\hat{j}+11\hat{k}$. Then the area of the parallelogram with diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is

(a) $4\sqrt{6}$ (b) $\frac{1}{2}\sqrt{21}$
(c) $\frac{\sqrt{6}}{2}$ (d) $\sqrt{6}$
(e) $\frac{1}{\sqrt{6}}$

53. If $|\vec{a}| = 3, |\vec{b}| = 1, |\vec{c}| = 4$ and $\vec{a}+\vec{b}+\vec{c} = 0$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to

(a) 13 (b) 26 (c) −29 (d) −13
(e) −26

54. If $|\vec{a}-\vec{b}| = |\vec{a}| = |\vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is equal to

(a) π/3 (b) 3π/4 (c) π/2 (d) 0
(e) π

55. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + 9\hat{j} + \mu\hat{k}$ are mutually orthogonal, then $\lambda + \mu$ is equal to

(a) 5 (b) −9 (c) −1 (d) 0
(e) −5

56. The solutions of $x^{2/5} + 3x^{1/5} - 4 = 0$ are

(a) 1, 1024 (b) −1, 1024
(c) 1, 1031 (d) −1024, 1
(e) −1, 1031

57. If the equations $x^2 + ax + 1 = 0$ and $x^2 - x - a = 0$ have a real common root b, then the value of b is equal to

(a) 0 (b) 1 (c) −1 (d) 2
(e) 3

58. If $\sin\theta - \cos\theta = 1$, then the value of $\sin^3\theta - \cos^3\theta$ is equal to

(a) 1 (b) −1 (c) 0 (d) 2
(e) −2

59. Two dice of different colours are thrown at a time. The probability that the sum is either 7 or 11 is

(a) $\frac{7}{36}$ (b) $\frac{2}{9}$ (c) $\frac{2}{3}$ (d) $\frac{5}{9}$
(e) $\frac{6}{7}$

60. $\frac{1}{9!} + \frac{1}{3!7!} + \frac{1}{5!5!} + \frac{1}{7!3!} + \frac{1}{9!}$ is equal to

(a) $\frac{2^9}{10!}$ (b) $\frac{2^{10}}{8!}$ (c) $\frac{2^{11}}{9!}$ (d) $\frac{2^{10}}{7!}$
(e) $\frac{2^8}{9!}$

61. The order and degree of the differential equation $(y''')^2 + (y'')^3 - (y')^4 + y^5 = 0$ is

(a) 3 and 2 (b) 1 and 2
(c) 2 and 3 (d) 1 and 4
(e) 3 and 5

62. $\int_{-2}^{2} |x|\,dx$ is equal to

(a) 0 (b) 1 (c) 2 (d) 4
(e) 1/2

63. $\int_{-1}^{0} \frac{dx}{x^2 + 2x + 2}$ is equal to

(a) 0 (b) π/4 (c) −π/4 (d) π/2
(e) π/2

64. If $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4} (3 - f(x))dx = 7$, then $\int_{-1}^{2} f(x)dx$ is

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

65. $\int \frac{xe^x}{(1+x)^2}\,dx =$

(a) $\frac{e^x}{1+x} + c$ (b) $\frac{e^x}{1+e^x} + c$
(c) $\frac{e^{2x}}{1+e^x} + c$ (d) $\frac{e^{-x}}{1+x} + c$
(e) $\frac{e^{-2x}}{1+x} + c$

66. The remainder when 2^{2000} is divided by 17 is

(a) 1 (b) 2 (c) 8 (d) 12
(e) 4

67. The coefficient of x^5 in the expansion of $(x + 3)^8$ is

(a) 1542 (b) 1512 (c) 2512 (d) 12
(e) 4

68. The maximum value of $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ is

(a) 5 (b) 11 (c) 10 (d) −1
(e) 2

69. The area of the triangle in the complex formed by z, iz and $z + iz$ is

(a) $|z|$ (b) $|\bar{z}|^2$
(c) $\frac{1}{2}|z|^2$ (d) $\frac{1}{2}|z + iz|^2$
(e) $|z + iz|$

70. Let $f : i \to j$ be a differentiable function. If f is even, then $f'(0)$ is equal to

(a) 1 (b) 2 (c) 0 (d) −1
(e) 1/2

71. The coordinate of the point dividing internally the line joining the points (4, −2) and (8, 6) in the ratio 7 : 5 is

(a) (16, 18) (b) (18, 16)
(c) $\left(\frac{19}{3}, \frac{8}{3}\right)$ (d) $\left(\frac{8}{3}, \frac{19}{3}\right)$
(e) (7, 3)

72. The area of the triangle formed by the points $(a, b + c)$, $(b, c + a)$, $(c, a + b)$ is
(a) abc (b) $a^2 + b^2 + c^2$
(c) $ab + bc + ca$ (d) 0
(e) $a(ab + bc + ca)$

73. If (x, y) is equidistant from $(a + b, b - a)$ and $(a - b, a + b)$, then
(a) $ax + by = 0$ (b) $ax - by = 0$
(c) $bx + ay = 0$ (d) $bx - ay = 0$
(e) $x = y$

74. The equation of the line passing through (a, b) and parallel to the line $\frac{x}{a}+\frac{y}{b}=1$ is
(a) $\frac{x}{a}+\frac{y}{b}=3$ (b) $\frac{x}{a}+\frac{y}{b}=2$
(c) $\frac{x}{a}+\frac{y}{b}=0$ (d) $\frac{x}{a}+\frac{y}{b}+2=0$
(e) $\frac{x}{a}+\frac{y}{b}=4$

75. If the points $(2a, a)$, $(a, 2a)$ and (a, a) enclose a triangle of area 18 square units, then the centroid of the triangle is equal to
(a) (4, 4) (b) (8, 8)
(c) (−4, −4) (d) $(4\sqrt{2}, 4\sqrt{2})$
(e) (6, 6)

76. The area of a triangle is 5 sq. units. Two of its vertices are (2, 1) and (3, −2). The third vertex lies on $y = x + 3$. The coordinates of the third vertex can be
(a) $\left(\frac{-3}{2}, \frac{-3}{2}\right)$ (b) $\left(\frac{3}{4}, \frac{-3}{2}\right)$
(c) $\left(\frac{7}{2}, \frac{13}{2}\right)$ (d) $\left(\frac{-1}{4}, \frac{1}{2}\right)$
(e) $\left(\frac{3}{2}, \frac{3}{2}\right)$

77. If $x^2 + y^2 + 2gx + 2fy + 1 = 0$ represents a pair of straight lines, then $f^2 + g^2$ is equal to
(a) 0 (b) 1 (c) 2 (d) 4
(e) 3

78. If θ is the angle between the pair of straight lines $x^2 - 5xy + 4y^2 + 3x - 4 = 0$, then $\tan^2\theta$ is equal to
(a) $\frac{9}{16}$ (b) $\frac{16}{25}$ (c) $\frac{9}{25}$ (d) $\frac{21}{25}$
(e) $\frac{25}{9}$

79. If $3\hat{i}+2\hat{j}-5\hat{k}=x(2\hat{i}-\hat{j}+\hat{k})+y(\hat{i}+3\hat{j}-2\hat{k})+z(-2\hat{i}+\hat{j}-3\hat{k})$, then
(a) $x = 1, y = 2, z = 3$ (b) $x = 2, y = 3, z = 1$
(c) $x = 3, y = 1, z = 2$ (d) $x = 1, y = 3, z = 2$
(e) $x = 2, y = 2, z = 3$

80. $\sin 15° =$
(a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}+1}{2\sqrt{2}}$
(c) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{1+\sqrt{3}}{\sqrt{2}}$
(e) $\frac{-(1+\sqrt{3})}{2\sqrt{2}}$

81. If $\vec{a}$ and $\vec{b}=3\hat{i}+6\hat{j}+6\hat{k}$ are collinear and $\vec{a}\cdot\vec{b}=27$, then $\vec{a}$ is equal to
(a) $3(\hat{i}+\hat{j}+\hat{k})$ (b) $\hat{i}+2\hat{j}+2\hat{k}$
(c) $2\hat{i}+2\hat{j}+2\hat{k}$ (d) $\hat{i}+3\hat{j}+3\hat{k}$
(e) $\hat{i}-3\hat{j}+2\hat{k}$

82. If $|\vec{a}|=13$, $|\vec{b}|=5$ and $\vec{a}\cdot\vec{b}=30$, then $|\vec{a}\times\vec{b}|$ is equal to
(a) 30 (b) $\frac{30}{25}\sqrt{233}$
(c) $\frac{30}{33}\sqrt{193}$ (d) $\frac{65}{23}\sqrt{493}$
(e) $\frac{65}{13}\sqrt{133}$

83. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, then r is equal to
(a) 69 (b) 41 (c) 51 (d) 61
(e) 49

84. Distance between two parallel lines $y = 2x + 4$ and $y = 2x - 1$ is
(a) 5 (b) $5\sqrt{5}$
(c) $\sqrt{5}$ (d) $\frac{1}{5}$
(e) $\frac{3}{\sqrt{5}}$

85. $({}^7C_0 + {}^7C_1) + ({}^7C_2 + {}^7C_3) + \ldots + ({}^7C_6 + {}^7C_7) =$
(a) $2^8 - 2$ (b) $2^7 - 1$
(c) 2^7 (d) $2^8 - 1$
(e) $2^7 - 2$

86. The coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$ is
(a) 17 (b) 19 (c) –17 (d) –19
(e) 20

87. The equation of the circle with centre (2, 2) which passes through (4, 5) is
(a) $x^2 + y^2 - 4x + 4y - 77 = 0$
(b) $x^2 + y^2 - 4x - 4y - 5 = 0$
(c) $x^2 + y^2 + 2x + 2y - 59 = 0$
(d) $x^2 + y^2 - 2x - 2y - 23 = 0$
(e) $x^2 + y^2 + 4x - 2y - 26 = 0$

88. The point in the xy-plane which is equidistant from (2, 0, 3), (0, 3, 2) and (0, 0, 1) is
(a) (1, 2, 3) (b) (–3, 2, 0)
(c) (3, –2, 0) (d) (3, 2, 0)
(e) (3, 2, 1)

89. Let $f : N \to N$ be such that $f(1) = 2$ and $f(x + y) = f(x)\, f(y)$ for all natural numbers x and y. If $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, then a is equal to
(a) 3 (b) 4 (c) 5 (d) 6
(e) 7

90. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then $n =$
(a) 3 (b) 4 (c) 8 (d) 9
(e) 10

91. Let $f : (-1, 1) \to (-1, 1)$ be continuous, $f(x) = f(x^2)$ for all $x \in (-1, 1)$ and $f(0) = 1/2$. Then the value of $4f\left(\frac{1}{4}\right)$ is
(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

92. $\lim_{x\to\infty} \sqrt{x^2+1} - \sqrt{x^2-1} =$
(a) –1 (b) 1 (c) 0 (d) 2
(e) 4

93. If f is differentiable at $x = 1$ and $\lim_{h\to 0} \frac{1}{h} f(1+h) = 5$, then $f'(1) =$
(a) 0 (b) 1 (c) 3 (d) 4 (e) 5

94. The maximum value of the function $2x^3 - 15x^2 + 36x + 4$ is attained at
(a) 0 (b) 3 (c) 4 (d) 2 (e) 5

95. If $\int f(x)\cos x\,dx = \frac{1}{2}\{f(x)\}^2 + c$, then $f\left(\frac{\pi}{2}\right)$ is
(a) c (b) $\frac{\pi}{2} + c$
(c) $c + 1$ (d) $2\pi + c$
(e) $c + 2$

96. $\int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x}\,dx =$
(a) $\pi(\sqrt{2}-1)$ (b) $\pi(\sqrt{2}+1)$
(c) $2\pi(\sqrt{2}-1)$ (d) $2\pi(\sqrt{2}+1)$
(e) $\frac{\pi}{\sqrt{2}+1}$

Top 10

CAREERS

Ethical hacking, as a career choice, is gaining popularity among students. While it was one of the popular choices last year, this year, it has emerged as a third most favourite choice, according to Mindler.

Defence and related fields have emerged as the most searched career options.

Mathematics, a popular traditional subject, is back in trend again.

	2017	2018
1.	Engineering	Maths
2.	Design	Computer Applications
3.	Media Communication	Ethical Hacking
4.	Management	Animations and Graphics
5.	Social Science	Architecture
6.	Ethical Hacking	Merchant Navy
7.	Computer Applications	Defense
8.	Hotel Management	Food and Agriculture
9.	Maths	Engineering
10.	Finance	Allied Medicine

97. $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}dx =$

(a) 2 (b) π (c) $\pi/4$ (d) 2π
(e) 0

98. $\lim_{x\to 0}\left(\frac{\int_0^{x^2}\sin\sqrt{t}dt}{x^2}\right) =$

(a) $\frac{2}{3}$ (b) $\frac{2}{9}$ (c) $\frac{1}{3}$ (d) 0
(e) $\frac{1}{6}$

99. The area bounded by $y = \sin^2 x$, $x = \pi/2$ and $x = \pi$ is
(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/8$ (d) $\pi/16$
(e) 2π

100. The differential equation of the family of curves $y = e^x(A\cos x + B\sin x)$, where A and B are arbitrary constants is
(a) $y'' - 2y' + 2y = 0$ (b) $y'' + 2y' - 2y = 0$
(c) $y'' + y'^2 + y = 0$ (d) $y'' + 2y'^2 - y = 0$
(e) $y'' - 2y' - 2y = 0$

101. The real part of $(i-\sqrt{3})^{13}$ is
(a) 2^{-10} (b) 2^{12} (c) 2^{-12} (d) -2^{-12}
(e) 2^{10}

102. $\lim_{x\to 0}\frac{1+x-e^x}{x^2} =$
(a) 1/2 (b) −1/2 (c) 1 (d) −1
(e) 0

103. $\int \frac{(\sin x+\cos x)(2-\sin 2x)}{\sin^2 2x}dx =$

(a) $\frac{\sin x+\cos x}{\sin 2x}+c$ (b) $\frac{\sin x-\cos x}{\sin 2x}+c$
(c) $\frac{\sin x}{\sin x+\cos x}+c$ (d) $\frac{\sin x}{\sin x-\cos x}+c$
(e) $\frac{\sin x-\cos x}{\sin x+\cos x}+c$

104. A plane is at a distance of 5 units from the origin, and perpendicular to the vector $2\hat{i}+\hat{j}+2\hat{k}$. The equation of the plane is
(a) $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})=15$ (b) $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})=15$
(c) $\vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k})=15$ (d) $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k})=15$
(e) $\vec{r}\cdot(2\hat{i}-\hat{j}+2\hat{k})=15$

105. $\frac{\sin A-\sin B}{\cos A+\cos B}$ is equal to
(a) $\sin\left(\frac{A+B}{2}\right)$ (b) $2\tan(A+B)$
(c) $\cot\left(\frac{A-B}{2}\right)$ (d) $\tan\left(\frac{A-B}{2}\right)$
(e) $2\cot(A+B)$

106. If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2} =$
(a) x (b) $-16x$ (c) $15x$ (d) $16x$
(e) $-15x$

107. The arithmetic mean of ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ is
(a) $\frac{2^n}{n+1}$ (b) $\frac{2^n}{n}$ (c) $\frac{2^{n-1}}{n+1}$ (d) $\frac{2^{n-1}}{n}$
(e) $\frac{2^{n+1}}{n}$

108. The variance of first 20 natural numbers is
(a) $\frac{399}{2}$ (b) $\frac{379}{12}$ (c) $\frac{133}{2}$ (d) $\frac{133}{4}$
(e) $\frac{169}{2}$

109. If S is a set with 10 elements and $A = \{(x, y) : x, y \in S, x \neq y\}$, then the number of elements in A is
(a) 100 (b) 90 (c) 80 (d) 150
(e) 45

110. A coin is tossed and a die is rolled. The probability that the coin shows head and the die shows 3 is
(a) 1/6 (b) 1/12 (c) 1/9 (d) 11/12
(e) 1/11

111. If $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$, then the sum of all the diagonal entries of A^{-1} is
(a) 2 (b) 3 (c) −3 (d) −4
(e) 4

112. Let $f(x)=\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$. If $x = -9$ is a root of $f(x) = 0$, then the other roots are

(a) 2 and 7 (b) 3 and 6
(c) 7 and 3 (d) 6 and 2
(e) 6 and 7

113. If $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}=0$, then x can be

(a) -2 (b) 2 (c) 14 (d) -14
(e) 0

114. If $A=\begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1}=\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then $x =$

(a) 2 (b) 1/2 (c) 1 (d) 3
(e) 0

115. If $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix}=ax^4+bx^3+cx^2+dx+e$,

then $5a + 4b + 3c + 2d + e$ is equal to

(a) 11 (b) -11 (c) 12 (d) -12
(e) 13

116. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}=$

(a) 1
(b) 0
(c) $(1 - a)(1 - b)(1 - c)$
(d) $a + b + c$
(e) $2(a + b + c)$

117. If $f(x)=\begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x(x-1) & (x-1)(x-2) & x(x-1) \end{vmatrix}$, then $f(50) =$

(a) 0 (b) 2 (c) 4 (d) 1
(e) 3

118. If $\Delta(x)=\begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$, then $\int_0^{\pi/2} \Delta(x)\,dx =$

(a) $-1/2$ (b) 1/2 (c) 1 (d) -1 (e) 0

119. The equation of the plane passing through the points (1, 2, 3), (−1, 4, 2) and (3, 1, 1) is

(a) $5x + y + 12z - 23 = 0$
(b) $5x + 6y + 2z = 23$
(c) $x + 6y + 2z = 13$
(d) $x + y + z = 13$ (e) $2x + 6y + 5z = 7$

120. In an arithmetic progression, if the k^{th} term is $5k + 1$, then the sum of first 100 terms is

(a) 50(507) (b) 51(506)
(c) 50(506) (d) 51(507)
(e) 52(506)

ANSWER KEY

1. (b)	**2.** (d)	**3.** (e)	**4.** (a)	**5.** (b)	**6.** (e)	**7.** (e)	**8.** (d)	**9.** (e)	**10.** (c)
11. (a)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (d)	**19.** (None)	
20. (a)	**21.** (a)	**22.** (c)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (c)	**27.** (b)	**28.** (a)	**29.** (c)
30. (b)	**31.** (c)	**32.** (c)	**33.** (a)	**34.** (e)	**35.** (b)	**36.** (b)	**37.** (b)	**38.** (a)	**39.** (d)
40. (b)	**41.** (b)	**42.** (c)	**43.** (None)		**44.** (a)	**45.** (d)	**46.** (b)	**47.** (d)	**48.** (a)
49. (d)	**50.** (b)	**51.** (c)	**52.** (a)	**53.** (d)	**54.** (a)	**55.** (b)	**56.** (d)	**57.** (c)	**58.** (a)
59. (b)	**60.** (a)	**61.** (a)	**62.** (d)	**63.** (b)	**64.** (e)	**65.** (a)	**66.** (a)	**67.** (b)	**68.** (c)
69. (c)	**70.** (c)	**71.** (c)	**72.** (d)	**73.** (d)	**74.** (b)	**75.** (b)	**76.** (c)	**77.** (b)	**78.** (c)
79. (c)	**80.** (a)	**81.** (b)	**82.** (e)	**83.** (b)	**84.** (c)	**85.** (c)	**86.** (d)	**87.** (b)	**88.** (d)
89. (a)	**90.** (d)	**91.** (b)	**92.** (c)	**93.** (e)	**94.** (d)	**95.** (c)	**96.** (a, e)	**97.** (c)	**98.** (d)
99. (b)	**100.** (a)	**101.** (None)		**102.** (b)	**103.** (b)	**104.** (c)	**105.** (d)	**106.** (b)	**107.** (a)
108. (d)	**109.** (b)	**110.** (b)	**111.** (e)	**112.** (a)	**113.** (a, d)	**114.** (b)	**115.** (b)	**116.** (b)	**117.** (a)
118. (a)	**119.** (b)	**120.** (a)							

For detailed solutions refer to MTG Kerala PET Chapterwise Solutions.

OLYMPIAD CORNER

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1. A positive integer is called monotonic if its digits in base 10, read from left, to right, are in nondecreasing order. Prove that for each $n \in N$, there exists an n-digit monotonic number which is a perfect square.

2. Let P be the intersection point of the diagonals AC and BD of the convex quadrilateral $ABCD$ in which $AB = AC = BD$. Let O and I be the circumcenter and incenter, respectively, of triangle ABP. Prove that if $O \neq I$, then lines OI and CD are perpendicular.

3. The incircle of the isosceles triangle ABC touches the legs AC and BC at points M and N, respectively. A line t is drawn tangent to minor arc MN, intersecting NC and MC at points P and Q, respectively. Let T be the intersection point of lines AP and BQ.

(a) Prove that T lies of $\overline{MN}$;

(b) Prove that the sum of the areas of triangles ATQ and BTP is smallest when t is parallel to line AB.

4. The feet of the angle bisectors of triangle ABC are X, Y, and Z. The circumcircle of triangle XYZ cuts off there segments from lines AB, BC, and CA. Prove that two of these segments' lengths add up to the third segment's length.

5. An exam paper consists of 5 multiple-choice questions, each with 4 different choices; 2000 students take the test, and each student chooses exactly one answer per question. Find the smallest value of n for which it is possible for the students' answer sheets to have the following property: among any n of the students' answer sheets, there exist 4 of them among which any two have at most 3 common answers.

6. Let m, n be positive integers such that $m \leq \frac{n^2}{4}$ and every prime divisor of m is less than or equal to n. Show that m divides $n!$.

7. Let p be a prime number such that $p \equiv 1 \pmod 4$. Evaluate

$$\sum_{k=1}^{p-1}\left(\left\lfloor \frac{2k^2}{p} \right\rfloor - 2\left\lfloor \frac{k^2}{p} \right\rfloor\right).$$

8. The real numbers a, b, c, x, y, z satisfy $a \geq b \geq c > 0$ and $x \geq y \geq z > 0$. Prove that

$$\frac{a^2x^2}{(by+cz)(bz+cy)} + \frac{b^2y^2}{(cz+ax)(cx+az)} + \frac{c^2z^2}{(ax+by)(ay+bx)}$$

is at least $\frac{3}{4}$.

9. If $g(x)$ and $h(x)$ are polynomials with real coefficients and $f(x) = g(x^3) + xh(x^3)$ is divisible by $x^2 + x + 1$, show that $g(x)$ and $h(x)$ are both divisible by $x - 1$.

10. In a triangle ABC, and A is twice angle B. Show that $a^2 = b(b + c)$.

11. M is an interior point of a triangle ABC. Bisectors of interior angles BMC, CMA, AMB intersect BC, CA, AB respectively at X, Y, Z. Show that AX, BY, CZ are concurrent; if P is the point of concurrence and

$$\frac{PA}{PX}\cdot\frac{PB}{PY}\cdot\frac{PC}{PZ} = 8$$

show that M is the circumcenter of ΔABC.

12. If $A_1, A_2, \ldots, A_n$ is an n - sided regular polygon such that $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$, find n, the number of sides of the polygon.

SOLUTIONS

1. Any 1-digit perfect square (namely, 1, 4, or 9) is monotonic, proving the claim for $n = 1$. We now assume $n > 1$. If n is odd, write $n = 2k - 1$ for an integer $k \leq 2$, and let

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$$x_k = \frac{\left(10^k+2\right)}{6} = 1\underbrace{66\ldots6}_{k-2}7 .$$

Then, $x_k^2 = \dfrac{10^{2k}+4\cdot10^k+4}{36} = \dfrac{10^{2k}}{36}+\dfrac{10^k}{9}+\dfrac{1}{9}$. (*)

Observe that $\dfrac{10^{2k}}{36} = 10^{2k-2}\left(\dfrac{72}{36}+\dfrac{28}{36}\right)$

$= 2\cdot10^{2k-2}+10^{2k-2}\cdot\dfrac{7}{9} = 2\underbrace{77\ldots7}_{2k-2}+\dfrac{7}{9}.$

Thus, the right-hand side of (*) equals

$$\left(2\underbrace{77\ldots7}_{2k-2}+\frac{7}{9}\right)+\left(\underbrace{11\ldots1}_{k}+\frac{1}{9}\right)+\frac{1}{9} = 2\underbrace{77\ldots7}_{k-2}\underbrace{88\ldots89}_{k-1},$$

an n-digit monotonic perfect square. If n is even, write $n = 2k$ for an integer $k \geq 1$, and let .

$$y_k = \frac{10^k+2}{3} = \underbrace{33\ldots3}_{k-1}4$$

Then, $y_k^2 = \dfrac{\left(10^{2k}+4\cdot10^k+4\right)}{9} = \dfrac{10^{2k}}{9}+4\cdot\dfrac{10^k}{9}+\dfrac{4}{9}$

$$= \left(\underbrace{11\ldots1}_{2k}+\frac{1}{9}\right)+\left(\underbrace{44\ldots4}_{k}+\frac{4}{9}\right)+\frac{4}{9} = \underbrace{11\ldots1}_{k}\underbrace{55\ldots5}_{k-1}6$$

an n-digit monotonic perfect square. This completes the proof.

2. We first prove a fact that is very helpful in proving that two segments are perpendicular: Given two segments $\overline{XY}$ and $\overline{UV}$, $\overline{XY}\perp\overline{UV}$, if and only if $UX^2 - XV^2 = UY^2 - YV^2$. To prove this result, let X' and Y' be the feet of the perpendiculars of X and Y, respectively, to line UV. Then $\overline{XY}\perp\overline{UV}$ if and only if $X' = Y'$, that is, if and only if

$UX' - X'V = UY' - Y'V$ (*)

where all distances are directed distances. Because $UX' + X'V = UV + UY' + Y'V$, equation (*) holds if and only if $UX'^2 - X'V'^2 = UY'^2 - Y'V^2$, or equivalently $UX^2 - XV^2 = UY^2 - YV^2$. Thus, it suffices to show that $DO^2 - CO^2 = DI^2 - CI^2$. Let $AB = AC = BD = p$, $PC = a$ and $PD = b$. Then, $AP = p - a$ and $BP = p - b$. Let R be the circumradius of triangle ABP. By the Power of a Point Theorem $pb = DP\cdot DB = DO^2 - R^2$. Likewise, $pa = CO^2 - R^2$. Hence, $DO^2 - CO^2 = p(b - a)$. Because triangle ABD is isosceles with $BA = BD$, and I lies on the bisector of angle ABD, $ID = IA$. Likewise, $IB = IC$. Let T be the point of tangency of the incircle of triangle ABC to side $\overline{AB}$. Then $AT = \dfrac{(AB+AP-BP)}{2} = \dfrac{(p+b-a)}{2}$ and $BT = \dfrac{(p+a-b)}{2}$.

Because $IT\perp AB$, $AI^2 - BI^2 = AT^2 - BT^2$. Putting the above arguments together, we find that $DI^2 = CI^2$
$= AI^2 - BI^2 = AT^2 - BT^2 = (AT + BT)(AT - BT)$
$= p(b - a) = DO^2 - CO^2$, as desired.

3. (a) The degenerate hexagon $AMQPNB$ is circumscribed about the incircle of triangle ABC. By Brianchon's Theorem, its diagonals $\overline{AP}$, $\overline{MN}$ and $\overline{QB}$ concur. Therefore, T lies on $\overline{MN}$.

One can also use a more elementary approach. Let R and S be the points of tangency of the incircle with sides $\overline{AB}$ and $\overline{PQ}$, respectively. Let $\overline{BQ}$ intersect $\overline{MN}$ and $\overline{SR}$ at T_1 and T_2, respectively. Because $\angle QMN = \angle PNM = \dfrac{\overset{\frown}{MN}}{2}$, we have $\sin\angle QMN = \sin\angle PNM = \sin\angle BNM$. Applying the Law of Sines to triangles MQT_1 and NBT_1 yields

$$\frac{QT_1}{QM} = \frac{\sin\angle QMN}{\sin\angle QT_1M} = \frac{\sin\angle BNM}{\sin\angle BT_1N} = \frac{BT_1}{BN} \text{ or } \frac{QT_1}{BT_1} = \frac{MQ}{BN}$$

Likewise, $\dfrac{QT_2}{BT_2} = \dfrac{SQ}{BR}$

By equal tangents, $BN = BR$ and $QM = QS$. Hence $\dfrac{QT_1}{BT_1} = \dfrac{QT_2}{BT_2}$.

Because T_1 and T_2 both lie on $\overline{BQ}$, we must have $T_1 = T_2$. Hence, $\overline{BQ}$, $\overline{MN}$, $\overline{SR}$ are concurrent. In exactly the same manner, we can prove that $\overline{AP}$, $\overline{MN}$, $\overline{SR}$ are concurrent. It follows that T lies on $\overline{MN}$.

(b) Let $\alpha = \angle CAB = \angle CBA$ and $\beta = \angle ACB$. Let $f = [AQT]+[BPT]=[ABQ]+[ABP]-2[ABT]$. Because triangle ABC is isosceles, $\overline{MN}\,||\,\overline{AB}$, implying that $[ABT]$ is constant. Hence, minimizing f is equivalent to minimizing $g = [ABQ] + [ABP]$. Note that $2g = AB(AQ + PB)\sin\alpha = AB(AB + PQ)\sin\alpha$ where $AQ + PB = AB + QP$ because quadrilateral $ABCD$, has an inscribed circle. Thus, it suffices to minimize PQ.

Let I be the incenter of triangle ABC, so that I is the excenter of triangle CPQ opposite C. Hence, $PC + CQ + QP = 2CM$ is constant. Let $\angle CPQ = p$ and $\angle CPQ = q$. Then, $p + q = \pi - \beta$ is constant as well. Applying the Law of Sines to triangle CPQ yields

$$\frac{CM}{PQ} = 1+\frac{CP}{PQ}+\frac{CQ}{PQ} = 1+\frac{\sin p+\sin q}{\sin\beta}$$

$$= 1+\frac{2\sin\dfrac{p+q}{2}\cos\dfrac{p-q}{2}}{\sin\beta}$$

Hence, it suffices to maximize $\cos\dfrac{p-q}{2}$. It follows that $[ATQ] + [BTP]$ is minimized when $p = q$, that is, when $\overline{PQ}\,||\,\overline{AB}$.

4. We use signed distances throughout the problem, where $a = BC$, $b = CA$, and $c = AB$ are positive. Also, let the specified circle intersect BC at X and P, CA at Y and Q, and AB at Z and R and let $x = PX$, $y = QY$, $z = RZ$. By the angle bisector theorem, $YA = \dfrac{bc}{(c+a)}$ and $AZ = \dfrac{bc}{(a+b)}$. Hence,

$$QA=\frac{bc}{(c+a)+y} \text{ and } AR=\frac{bc}{(a+b)-z}.$$

Applying the Power of a Point Theorem to point A and the specified circle gives

$$\frac{bc}{c+a}\left(\frac{bc}{c+a}+y\right)=\frac{bc}{a+b}\left(\frac{bc}{a+b}-z\right).$$

After multiplying through out by $\frac{a}{bc}$ and rearranging, we obtain

$$\frac{a}{c+a}y+\frac{a}{a+d}z=\frac{abc}{(a+b)^2}-\frac{abc}{(c+a)^2}.$$

Similarly, we have

$$\frac{b}{a+b}z+\frac{b}{b+c}x=\frac{abc}{(b+c)^2}-\frac{abc}{(a+b)^2}.$$

$$\frac{c}{b+c}x+\frac{c}{c+a}y=\frac{abc}{(c+a)^2}-\frac{abc}{(b+c)^2}.$$

When we add these three equations, we get $x + y + z = 0$. It follows that two of the quantities x, y, z are of the same sign and that the third is of the other sign. Hence, the sum of the absolute values of the former two quantities equals the absolute value of the latter, which is what we wished to prove.

5. First we prove that $n \geq 25$. Let 1, 2, 3, 4 denote the four different choices of each problem. Represent each student's answer sheet by an ordered 5-tuple $(a_1, a_2, a_3, a_4, a_5)$, $a_i \in \{1, 2, 3, 4\}$, where the student's answer to problem i is a_i. We say that two answer sheets are of the same type if their corresponding 5-tuples belong to a set of the form

$$\{(k, a_2, a_3, a_4, a_5) | k \in \{1, 2, 3, 4\}\},$$

where $a_2, a_3, a_4, a_5 \in \{1, 2, 3, 4)\}$. Since there are 256 such sets and $2000 = 256 \times 7 + 208$, at least eight answer sheets are of the same type by the pigeonhole Principle. Among the 1992 remaining answer sheets, again some eight are of the same type. Finally, among the 1984 remaining answer sheets, another eight are of the same type. Consider the set A of these 24 answer sheets. Given any four answer sheets in A, two of them must be of the same type, that is, their solutions for the last 4 problems are identical. This violates the assumption that there are 4 answer sheets in A, among which any two have at most 3 common answers. Hence, $n \geq 25$.

Now we show that $n = 25$ is indeed attainable. Define the set $S = \{(a_1, a_2, a_3, a_4, a_5) | \sum_{i=1}^{5} a_i \equiv 0 \pmod 4, a_i \in \{1, 2, 3, 4\}\}$. Then $|S| = 4^4 = 256$, and any two answer sheets have at most 3 common answers if their corresponding 5-tuples are distinct elements of S. Pick any 250 elements of S and assume that exactly eight students turn in answer sheets that correspond to each of these 250 5-tuples. Among any $25 > 3.8$ answer sheets, there are four whose corresponding 5-tuples are distinct elements in S, and they satisfy the given conditions of the problem.

Therefore, the answer is $n = 25$.

6. It suffices to show that $p^k n!$ for all primes p and integers $k \geq 1$ such that $p^k m$. If $k = 1$, we are done because we are given that $p \leq n$ and hence $p|n!$. Otherwise, because $m \leq \frac{n^2}{4}$, we have $p^k \leq \frac{n^2}{4}$. Thus, $n \geq 2\sqrt{p^k}$. If $n \geq kp$, then at least k of the numbers 1, 2, ..., n are multiples of p, implying that $p^k|n!$. Hence, it suffices to show that $2\frac{\sqrt{p^k}}{p} \geq k$ or equivalently that

$$p^{\frac{(k-2)}{2}} \geq \frac{k}{2}, \qquad (*)$$

because this implies $n \geq 2\sqrt{p^k} \geq kp$.

Indeed, we can do so for most values p and k. If $k = 2$, then (*) reads $1 \geq 1$. If $k \geq 4$, then Bernoulli's inequality implies that $p^{\frac{(k-2)}{2}} = (1+(p-1))^{\frac{(k-2)}{2}} \geq 1+\frac{k-2}{2}(p-1) \geq \frac{k}{2}$, as desired. Finally, if $k = 3$, then (*) is clearly true unless $p = 2$.

If indeed $k = 3$ and $p = 2$, then $m \geq 8$, $n \geq 6$ and $n!$ is certainly divisible by 8. Thus, we have shown that $p^k|n!$ in all cases, completing the proof.

7. For all real x, let $\{x\} = x - [x] \in [0, 1)$. Writing

$$\left\lfloor \frac{2k^2}{p} \right\rfloor = \frac{2k^2}{p} - \left\{\frac{2k^2}{p}\right\} \text{ and } \left\lfloor \frac{k^2}{p} \right\rfloor = \frac{k^2}{p} - \left\{\frac{k^2}{p}\right\},$$

we find that $\left\lfloor \frac{2k^2}{p} \right\rfloor - 2\left\lfloor \frac{k^2}{p} \right\rfloor = 2\left\{\frac{k^2}{p}\right\} - \left\{\frac{2k^2}{p}\right\}$

When $\{x\} < \frac{1}{2}$, $2\{x\} - 2\{x\} = 2\{x\} - 2\{x\} = 0$.

When $\{x\} \geq \frac{1}{2}$, $2\{x\} - \{2x\} = 2\{x\} - (2\{x\} - 1) = 1$. Therefore, the desired sum equals the number $\propto$ of k in $[1, p - 1]$ such that $\left\{\frac{k^2}{p}\right\} \geq \frac{1}{2}$, or equivalently, the number of nonzero residues k modulo p such that k^2 is congruent to some number in $\left[\frac{(p+1)}{2}, p-1\right]$ modulo p.

Because p is a prime congruent to 1 modulo 4, it is well known that $-1 \equiv d^2 \pmod p$ for some integer d. Partition the nonzero residues modulo p into $\frac{p-1}{2}$ pairs of the form $\{a, da\}$ so that $a^2 \equiv -(da)^2 \pmod p$ in each pair. Thus, exactly one residue in each pair has a square congruent to some number in $\left[\frac{p+1}{2}, p-1\right]$, for a total of $\frac{p-1}{2}$ such residues. It follows that the given sum equals $\frac{p-1}{2}$.

8. Denote the left-hand side of the given inequality by S. Because $a \geq b \geq c$ and $x \geq y \geq z$ by the rearrangement inequality we have $bz + cy \leq by + cz$ so
$(by + cz)(bz + cy) \leq (by + cz)^2 \leq 2((by)^2 + (cz)^2)$.
Setting $\alpha = (ax)^2$, $\beta = (by)^2$, $\gamma = (cz)^2$, we obtain

$$\frac{a^2x^2}{(by+cz)(bz+cy)} \geq \frac{a^2x^2}{2\left((by)^2+(cz)^2\right)} = \frac{\alpha}{2(\beta+\gamma)}.$$

Adding to this the two analogous inequalities for the other summands, we find that

$$S \geq \frac{1}{2}\left(\frac{\alpha}{\beta+\gamma}+\frac{\beta}{\gamma+\alpha}+\frac{\gamma}{\alpha+\beta}\right).$$

By the Cauchy-Schwarz inequality,

$$\left(\frac{\alpha}{\beta+\gamma}+\frac{\beta}{\gamma+\alpha}+\frac{\gamma}{\alpha+\beta}\right)(\alpha(\beta+\gamma)+\beta(\gamma+\alpha)+\gamma(\alpha+\beta))$$

is at least $(\alpha + \beta + \gamma)^2$, which in turn equals

$$\frac{1}{2}\left((\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2\right)+3(\alpha\beta+\beta\gamma+\gamma\alpha)$$

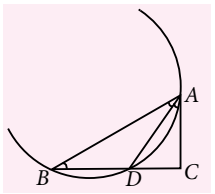
$$\geq \frac{3}{2}(2\alpha\beta+2\beta\gamma+2\gamma\alpha).$$

Therefore,

$$S \geq \frac{1}{2}\left(\frac{\alpha}{\beta+\gamma}+\frac{\beta}{\gamma+\alpha}+\frac{\gamma}{\alpha+\beta}\right) \geq \frac{1}{2}\frac{(\alpha+\beta+\gamma)^2}{(2\alpha\beta+2\beta\gamma+2\gamma\alpha)} \geq \frac{3}{4},$$ as desired.

9. **1st solution:** If $f(x) = g(x^3) + xh(x^3)$ is divisible by $x^2 + x + 1$, we can write $f(x) = q(x)(x^2 + x + 1)$. Let ω be a non - real cube root of unity. Then $f(\omega) = q(\omega)(\omega^2 + \omega + 1) = 0$ and $f(\omega^2) = q(\omega^2)(\omega + \omega^2 + 1) = 0$. Therefore $g(1) + \omega h(1) = 0$ and $q(1) + \omega^2 h(1) = 0$. So, it follows that $g(1) = h(1) = 0$; therefore both $g(x)$ and $h(x)$ are divisible by $x - 1$.
2nd solution: we can write $g(x) = (x - 1)g'(x) + g(1)$.
Then, $g(x^3) = (x^3 - 1)\, g'(x^3) + g(1)$. Similarly, we can write $h(x^3) = (x^3 - 1)\, h'(x^3) + h(1)$. Therefore, $f(x) = (x^3 - 1)\, g'(x^3) + xh'(x^3)) + g(1) + xh(1)$.
Since $x^2 + x + 1$ divides both $f(x)$ and $x^3 - 1$, it also divides $g(1) + xh(1)$. Therefore $g(1) = h(1) = 0$.

10. **1st solution:** Let AD be the internal bisector of $\angle A$, meeting BC at D. Then $\frac{BD}{DC} = \frac{AB}{AC}$; adding 1 to both sides we get $\frac{BD}{DC} = \frac{AB+AC}{AC}$ ...(i)

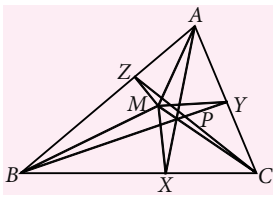


Since, $\angle CAD = \angle ABD$, AC is tangent to the circumcircle of ΔABD. Therefore, $BC.DC = AC^2$. ...(ii)
Now (i) × (ii) $\Rightarrow BC^2 = AC(AB + AC)$.
2nd solution: $A = 2B \Rightarrow C = \pi - 3B$ $\sin C = \sin 3B$. Therefore,
$\sin^2 A = \sin^2 2B = 2\sin B\cos B\sin 2B$
$= \sin B(\sin B + \sin 3B) = \sin B(\sin B + \sin C)$
$\Rightarrow \quad a^2 = b(b + c)$.

11. Since MX is the internal bisector of $\angle BMC$, $\frac{BX}{XC} = \frac{MB}{MC}$; similarly, $\frac{CY}{YA} = \frac{MC}{MA}$ and $\frac{AZ}{ZB} = \frac{MA}{MB}$.

Therefore, $\frac{BX}{XC}.\frac{CY}{YA}.\frac{AZ}{ZB} = 1$.

Hence by Ceva's theorem, AX, BY, CZ are concurrent.
Let $\Delta BPC = t_1$, $\Delta APC = t_2$ and $\Delta APB = t_3$.



Then,

$$\frac{PA}{PX} = \frac{\Delta APB}{\Delta BPX} = \frac{\Delta APC}{\Delta PXC} = \frac{\Delta APB + \Delta APC}{\Delta BPX + \Delta PXC} = \frac{t_2 + t_3}{t_1}.$$

Similarly, $\frac{PB}{PY} = \frac{t_3 + t_1}{t_2}$ and $\frac{PC}{PZ} = \frac{t_1 + t_2}{t_3}$.

Now,

$$8 = \frac{(t_2 + t_3)(t_3 + t_1)(t_1 + t_2)}{t_1 t_2 t_3} \geq \frac{2\sqrt{t_2 t_3}\,2.\sqrt{t_3 t_1}.2\sqrt{t_1 t_2}}{t_1 t_2 t_3}$$

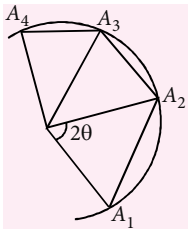
(Equality holds only when $t_1 = t_2 = t_3 = 8$)
Thus, $t_1 = t_2 = t_3$. Now, $\frac{BX}{XC} = \frac{\Delta ABX}{\Delta ACX}$

$$= \frac{\Delta PBX}{\Delta PCX} = \frac{\Delta ABX - \Delta PBX}{\Delta ACX - \Delta PCX} = \frac{t_3}{t_2} = 1$$

i.e., $BX = XC$. Since MB is the bisector of $\angle BMC$, it follows that $MB = MC$. Similarly $MC = MA$. Therefore, M is the circumcenter. (Note that P is the centroid).

12. We can assume that the radius of the circumcircle of the polygon is 1. Let 2θ be the angle subtended by any side of the polygon at the center of the circumcircle. We know, when a segment of the circle subtends an angle ϕ at the center, its length is $2\sin\frac{\phi}{2}$. Therefore, $A_1A_2 = 2\sin\theta$, $A_1A_3 = 2\sin 2\theta$ and $A_1A_4 = 2\sin 3\theta$. Now by hypothesis,

$$\frac{1}{\sin\theta} = \frac{1}{\sin 2\theta} + \frac{1}{\sin 3\theta}$$

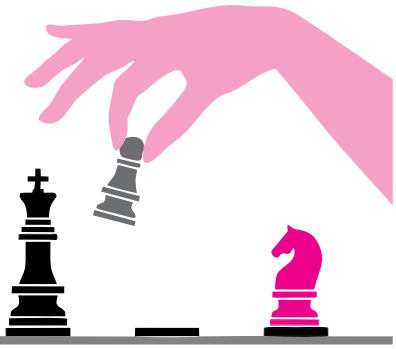


$$\Rightarrow \quad \frac{1}{\sin\theta} = \frac{\sin 3\theta + \sin 2\theta}{\sin 2\theta \sin 3\theta}$$

$\Rightarrow \quad \sin 3\theta + \sin 2\theta = 2\sin 3\theta\cos\theta$
$\qquad\qquad = \sin 4\theta + \sin 2\theta$
$\Rightarrow \quad \sin 3\theta + \sin 4\theta$
Since $3\theta, 4\theta \in [0,\pi]$, we get $3\theta + 4\theta = \pi = n\theta$
$\Rightarrow n = 7$.

Challenging PROBLEMS

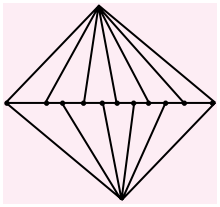
Permutations and Combinations



1. We call a number, a *Y*-number, if its middle point digit is larger than any other digit. For eg. 254 is a *Y*-number. How many 3-digit *Y*-numbers are there?

(a) 120 (b) 240 (c) 125 (d) 250

2. How many traingles are in the figure?



(a) 15
(b) 21
(c) 36
(d) 72

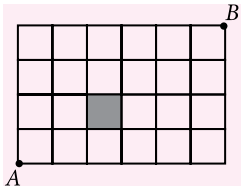
3. The number of pairs of diagonals of a regular decagon which are parallel is

(a) 15 (b) 30 (c) 45 (d) 90

4. 1000 unit cubes ($1 \times 1 \times 1$ cubes) are glued together to form a $10 \times 10 \times 10$ cube. The maximum number of unit cubes visible from a single point in space are

(a) 271 (b) 315 (c) 729 (d) 912

5. In the grid at right, we wish to go from corner *A* to corner *B* moving only up and right one unit at a time. The number of paths that include an edge of the shaded square is



(a) 120 (b) 135 (c) 145 (d) 150

6. Mr. *X* can remember 10-digit numbers. If the first four digits are either exactly the same as the next four digits or the last four digits of the number. The number of 10-digit numbers that Mr. *X* can remember is

(a) 1799910 (b) 1799909
(c) 1699910 (d) 1699909

7. The number of non-empty subsets of {1, 2, 3, ..., 12} having the property that the sum of the largest element and the smallest element is 13 is

(a) 1165 (b) 1245
(c) 1365 (d) 2540

8. Consider the sequence 1, 4, 5, 16, 17, 20, 21, 64, 65, ... which is formed by including only positive integers that can be expressed as the sum of distinct powers of 4. The 50^{th} term of this sequence is

(a) 1284 (b) 2540
(c) 3142 (d) 4128

9. The size of the largest subset S of {1, 2, 3, ... 50} such that no pair of distinct elements of *S* has a sum divisible by 7 is

(a) 12 (b) 24 (c) 23 (d) 25

10. A subset *A* of the set of integers from 1 to 100, inclusive has the property that no two elements of *A* sum to 125. The maximum possible number of elements in *A* is

(a) 60 (b) 62 (c) 64 (d) 66

11. The number of ways 3 teachers and 8 students can sit in 11 chairs, if there must be at least 2 students between each pair of teachers is

(a) ${}^7C_3 \cdot 8!\ 3!$ (b) ${}^7C_2 \cdot 7!\ 4!$
(c) ${}^7C_3 \cdot 9!\ 2!$ (d) ${}^7C_4 \cdot 11!$

12. The parking lot outside a building has 12 parking spaces. Compact cars can easily fit within a single space, but XUV cars take up two spaces; the number of different ways the parking lot can be filled is

(a) 233 (b) 235 (c) 242 (d) 254

13. Simplify,

$$\frac{19}{99}+\frac{(19)(18)}{(99)(98)}+\frac{(19)(18)(17)}{(99)(98)(97)}+.....+\frac{19!}{(99)(98)...(81)}$$

(a) $\frac{19}{81}$ (b) $\frac{19}{99}$ (c) $\frac{19}{98}$ (d) $\frac{19}{82}$

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14. The number of subsets with three elements that can be formed from the set {1, 2, ..., 20} so that 4 is a factor of the product of the three numbers in the subset is

(a) 700 (b) 795 (c) 800 (d) 825

15. A number of elliptic curves are drawn on the plane, any two of them intersecting in 4 points and no three of the curves are concurrent. The number of regions the plane is divided, if n ellipses are drawn is

(a) $2n^2 - 2n + 2$ (b) $4n^2 - 2$

(c) $2n^2 + 2n + 2$ (d) $2n^2$

16. Evaluate : $\sum_{r=0}^{\infty} \frac{{}^{n}C_r}{{}^{2n-1}C_r}$.

(a) 1 (b) 2 (c) 1/2 (d) 3

17. The number of positive integer n (< 17) for which $n! + (n+1)! + (n+2)!$ is an integral multiple of 49 is

(a) 3 (b) 4 (c) 5 (d) 6

18. n elements $a_1, a_2, ..., a_n$ are permuted among themselves. Now, let f_n denote the number of permutations in which no element remain fixed, *i.e.*, every element changes its position and let g_n denote the number of permutations where only one element remains fixed and remaining $(n-1)$ elements change their position, then $|f_n - g_n| =$

(a) $n - 1$ (b) 1

(c) 2 (d) $(n + 1)$

SOLUTIONS

1. (b) : We have following 3 cases:

(i) xyx form numbers $(x \neq 0)$

(ii) xyz form numbers $(x \neq z, z \neq 0)$

(iii) $xy0$ form numbers $(x \neq 0, y \neq 0)$

Case (i) : Numbers are ${}^{9}C_2$.

Case (ii) : Numbers are (xyz) and $(zyx) = 2 \times {}^{9}C_3$

Case (iii) : Numbers are ${}^{9}C_2$

So, total numbers $= {}^{9}C_2 + 2 \times {}^{9}C_3 + {}^{9}C_2 = 240$

2. (c) : Case (i) : Triangles containing top point. (Base is the middle line and we choose 2 out, then 7 lines emanating from top point). So, ${}^{7}C_2 = 21$

Case (ii) : Triangle containing bottom point.

Similarly, as before $= {}^{6}C_2 = 15$

Hence, total number of triangles $= 21 + 15 = 36$

3. (c) : Consider a decagon $ABCDEFGHIJ$.

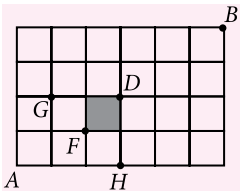
Fix vertex A. Diagonals BJ, CI, DH and EG are parallel to each other. There are 5 such sets of 4 parallel diagonals, since the 10 vertices can be paired into 5 sets. In each set we have ${}^{4}C_2 = 6$ pairs. So, in total 30 pairs in 5 sets.

Now fix side AB, then CJ, DI, EH are set of 3 parallel diagonals. This type is also 5 sets. So, total pairs of these types $= {}^{3}C_2 \times 5 = 15$

Hence, total pairs $= 30 + 15 = 45$

4. (a) : When we look from a point in space we can see 3 faces, the number of unit cubes visible is maximum. In that case, the $9 \times 9 \times 9$ cube that is behind the 3 visible faces is hidden from our view. Therefore, the number of visible cubes $= 10 \times 10 \times 10 - 9 \times 9 \times 9 = 271$

5. (c) : A path from $A \to B$ that includes an edge of the shaded square must pass through exactly one of the points F, G or H in the diagram.



Case (i) : Passing through H. Then it must pass through D. There is only 1 path from $A \to H \to D$ and ${}^{5}C_3 = 10$ from $D \to B$. So, total $A \to B \equiv 10 \times 1 = 10$

Case (ii) : Passing through G and then passing through side of shaded square to D.

From $A \to G \equiv {}^{3}C_1 = 3$ paths and ${}^{5}C_3 = 10$ from $D \to B$. So, total $A \to B = 3 \times 10 = 30$.

Case (iii) : Passing through F. $A \to F \equiv {}^{3}C_2 = 3$ and $F \to B \equiv {}^{7}C_4 = 35$. So, total $A \to B = 3 \times 35 = 105$

Hence, overall $10 + 30 + 105 = 145$ paths.

6. (a) : Let A be the set of 10-digit numbers where the first 4 digits are same as the next 4 digits and B be the set of 10 digit numbers where the first 4 digits are same as the last 4 digits. Then

$n(A) = (9000) \times (100) = 900{,}000$ and similarly, $n(B) = 900{,}000$

Now, $n(A \cap B) = 9 \times 10$ (numbers of form abababab)

$= 90$

So, $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 1799910$

7. (c) : Let t be the smallest element, so largest element is $13 - t$ *i.e.*, $t \leq 13 - t \Rightarrow t \leq 6$. The set can then contain any combination of elements from $(t + 1)$ to $(12 - t)$. There are $12 - 2t$ such elements, so there are $2^{(12-t)-(t+1)+1} = 2^{12-2t}$ such combinations.

So, the total number of subsets are

$$\sum_{t=1}^{6} 2^{12-2t} = 1024 + 256 + 64 + 16 + 4 + 1 = 1365$$

8. (a) : There is a one-one correspondence between the numbers in the sequence and the positive integers written in binary. We simply read the binary integer as an integer in base 4. Since, each digit is 0 or 1, the resulting base 4 number will be the sum of distinct

powers of 4. So, since 50 is 110010 in base 2, the 50^{th} number in the base 4 number 110010 is $4^5 + 4^4 + 4^1 = 1284$

9. (c) : Consider all numbers {1, 2, ..., 50} modulo 7, then we find that there are 7 numbers congruent to 0, 2, 3, 4, 5, 6 (modulo 7) and 8 numbers congruent 1 (mod 7). Then to maximize the number of elements in S, we take the 8 numbers congruent to 1 (mod 7), the 7 numbers congruent to 2 (mod 7) and 7 numbers congruent to 3 (mod 7). We can further add one number congruent to 0 (mod 7). So, S contains maximum $8 + 7 + 7 + 1 = 23$ elements.

10. (b) : If two numbers sum to 125, then they must be {25, 100}, {26, 99} ,...., {62, 63}. There are 38 such pairs and A can contain atmost one element from each pair. The subset A can also contain any combination of remaining elements 1, 2, 3, ..., 24. So, A can contain a max. of $38 + 24 = 62$ elements.

11. (a) : According to question, for minimum condition to be satisfied we must have this form TSSTSST (T = teacher, S = student). We still have 4 S to place in 4 spaces *i.e.* it is equivalent to solve $a + b + c + d = 4$ in non-negative integers, *i.e.* ${}^7C_3$ solutions.
Now, we can place the 8 S and 3 T to the respective positions in 8! and 3! ways.
Hence, total ways $= {}^7C_3 \cdot 8! \cdot 3!$

12. (a) : Let C_n be the number of ways that we can fill the parking lot with n spaces then,
$C_n = C_{n-1} + C_{n-2}$ with $C_1 = 1$, $C_2 = 2$
So, $C_{12} = 233$

13. (a) : Notice the following identity, which can be proved inductively,

$$\frac{a}{b}+\frac{a(a-1)}{b(b-1)}+....+\frac{a!}{b(b-1)....(b-(a-1))}=\frac{a}{b-a+1}$$

14. (b) : There are in all ${}^{20}C_3$ subsets. All of these, when multiplied will have 4 as a factor, except in two cases.
Case (i) : All elements are odd $= {}^{10}C_3$
Case (ii) : 2 elements are odd and 1 element is even but not multiple of 4 $= {}^{10}C_2 \times {}^5C_1$
Hence, total number of subsets
$= {}^{20}C_3 - ({}^{10}C_3) - ({}^{10}C_2 \times {}^5C_1) = 795$

15. (a) : One ellipse divides the plane into 2 regions, two ellipses divide into 6 regions and 3 ellipse into 14 regions. Let $F(n)$ be the number of regions then
$F(1) = 2$, $F(2) = 6$, $F(3) = 14$ and further
$F(n + 1) = F(n) + 4n$ *i.e.*, $F_n = F_{n-1} + 4(n - 1)$
Simplifying, $F_n = 2+4\left(\frac{(n-1)n}{2}\right)=2n^2-2n+2$

16. (b) : Notice that $\frac{{}^nC_r}{{}^{2n}C_r}-\frac{{}^nC_{r+1}}{{}^{2n}C_{r+1}}=\frac{1}{2}\left[\frac{{}^nC_r}{{}^{2n-1}C_r}\right]$

Hence, $\frac{{}^nC_r}{{}^{2n-1}C_r}=2\left[\frac{{}^nC_r}{{}^{2n}C_r}-\frac{{}^nC_{r+1}}{{}^{2n}C_{r+1}}\right]$ (consecutive terms)

i.e., $\sum_{r=0}^{\infty}\frac{{}^nC_r}{{}^{2n-1}C_r}=2\left[\frac{{}^nC_0}{{}^{2n}C_0}\right]=2$

17. (c) : $n! + (n + 1)! + (n + 2)! = n!(n + 2)^2$
$\Rightarrow$ either 7 divides $(n + 2)$ or 49 divides $n!$
So, only possibilities of n (< 17) are 5, 12, 14, 15, 16.

18. (b) : Notice that f_n is de-arrangement.
So, $f_n = (n - 1)(f_{n-1} + f_{n-2})$ and $g_n = n \cdot f_{n-1}$
And, $g_{n+1} - f_{n+1} = (n + 1)f_n - n(f_n + f_{n-1})$
$= f_n - g_n = (-1)(g_n - f_n) = (-1)^2 (g_{n-1} - f_{n-1}) = ...$
$= (-1)^n (g_1 - f_1)$, $(g_1 = 1, f_1 = 0) = (-1)^n$
Hence, $|f_n - g_n| = 1$.

COMIC CAPSULE

Read this. It will change you.
Wow..
Read this. It will change you.
Meh. It was alright, I guess.
How could anyone not be affected by this book?

MATHS MUSING

SOLUTION SET-185

1. (d): Let $z = r_1e^{i\theta}$ and $w = r_2e^{i\phi} \Rightarrow \bar{z} = r_1e^{-i\theta}$

Given, $|zw| = 1 \Rightarrow |r_1e^{i\theta}\cdot r_2e^{i\phi}| = 1 \Rightarrow r_1r_2 = 1$ ...(i)

and $\arg(z) - \arg(w) = \frac{\pi}{2} \Rightarrow \theta - \phi = \frac{\pi}{2}$ ...(ii)

Now, $\bar{z}w = r_1e^{-i\theta}\cdot r_2e^{i\phi} = r_1r_2e^{-i(\theta-\phi)}$

$= 1\cdot e^{-\pi/2}$ [from (i) and (ii)]

$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2} = -i$

2. (c): The given relation is possible, when

$a - \frac{a^2}{3} + \frac{a^3}{9} + ... = 1 + b + b^2 + ...$

Also, $-1 \le a - \frac{a^2}{3} + \frac{a^3}{9} + ... \le 1$

and $-1 \le 1 + b + b^2 + ... \le 1 \Rightarrow |b| < 1 \Rightarrow |a| < 3$

and $\frac{a}{1+\frac{a}{3}} = \frac{1}{1-b} \Rightarrow \frac{3a}{a+3} = \frac{1}{1-b}$, there are infinitely many solutions.

$\Rightarrow$ $3a - 3ab = a + 3 \Rightarrow 2a - 3ab = 3$

$\Rightarrow$ $b = \frac{2a-3}{3a}$ and $a = \frac{3}{2-3b}$

3. (c): Let P and Q be point on the lines, then

$P(-2 + 2s, -6 + 3s, 34 - 10s)$, $Q(-6 + 4t, 7 - 3t, 7 - 2t)$

The d.r's of PQ are $(-4 + 4t - 2s, 13 - 3t - 3s, -27 - 2t + 10s)$

PQ is perpendicular to the two lines with d.r's 2, 3, –10 and 4, –3, –2.

$\therefore$ $2(-4 + 4t - 2s) + 3(13 - 3t - 3s) - 10(-27 - 2t + 10s) = 0$,

$4(-4 + 4t - 2s) - 3(13 - 3t - 3s) - 2(-27 - 2t + 10s) = 0$

$\Rightarrow$ $113s - 19t = 301$, $29t - 19s = 1$

On solving, we get $s = 3$, $t = 2$, $P = (4, 3, 4) = (a, b, c)$

$\Rightarrow$ $a + b + c = 11$.

4. (d): $C_0 + C_1x + C_2x^2 + ... + C_nx^n = (1 + x)^n$

$C_0x^n + C_1x^{n-1} + C_2x^{n-2} + ... + C_rx^{n-r} + ... + C_n = (x + 1)^n$

Multiplying above two and taking the coefficients of x^{n-r}, we get

$C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ... C_{n-r}C_n$ = coefficient of x^{n-r} in $(1 + x)^n(x + 1)^n = (1 + x)^{2n}$ which is $\binom{2n}{n-r}$.

5. (b): $f'(x) = (x - 1)(x - 2)^2[2(x - 2) + 3(x - 1)]$

$= (x - 1)(5x - 7)(x - 2)^2$

$f'(x) = 0 \Rightarrow x = 1, \frac{7}{5}, 2$

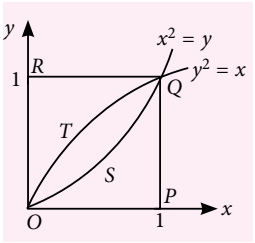
$x = 2$ is inflection point, $x = 1$ is maximum point and $x = \frac{7}{5}$ is minimum point.

6. (a): A = the event of (x, y) belonging to the area $OTQPO$

B = the event of (x, y) belonging to the area $OSQRO$

$P(A) = \frac{ar(OTQPO)}{ar(OPQRO)}$

$\frac{\int_0^1\sqrt{x}dx}{1\times1} = \left[\frac{2}{3}x^{3/2}\right]_0^1 = \frac{2}{3}$



$P(B) = \frac{ar(OSQRO)}{ar(OPQRO)} = \frac{\int_0^1\sqrt{y}dy}{1\times1} = \frac{2}{3}$

$P(A \cap B) = \frac{ar(OTQS)}{ar(OPQRO)} = \frac{\int_0^1\sqrt{x}dx - \int_0^1\sqrt{x^2}dx}{1\times1} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

$P(A) + P(B) = \frac{2}{3} + \frac{2}{3} \neq 1$

So, A and B are not exhaustive.

$P(A)\cdot P(B) = \frac{2}{3}\cdot\frac{2}{3} = \frac{4}{9} \neq P(A \cap B)$

So, A and B are not independent.

$P(A \cup B) = 1$, $P(A) + P(B) = \frac{2}{3} + \frac{2}{3} \neq P(A \cup B)$

So, A and B are not mutually exclusive.

7. (c): Given $f(1) = 8$ and $f(2) = 32$

$f(x + y) - axy = f(x) + by^2$ ...(i)

Putting $x = 1$ and $y = 1$ in (i), we get

$f(1 + 1) - a = f(1) + b$

$\Rightarrow$ $f(2) = a + f(1) + b \Rightarrow 32 = a + b + 8$

$\therefore$ $a + b = 24$ ...(ii)

Putting $x = 2$, $y = -1$ in (i), we get

$f(1) + 2a = f(2) + b \Rightarrow 2a - b = 24$ ...(iii)

On solving (ii) & (iii), we get, $a = 16$, $b = 8$

8. (a): $f(x + y) - 16xy = f(x) + 8y^2$

$\Rightarrow$ $f(x + y) - f(x) = 16xy + 8y^2$

$f'(x) = \lim_{h\to0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to0}\frac{16xh+8h^2}{h} = 16x$

$\therefore$ $f(x) = 8x^2$

9. (2): The centres of the circles $C_1(1, 0)$ and $C_2(2, 1)$ and radii are $r_1 = 1$, $r_2 = 2$. The common tangents meet at T.

$\therefore$ $T \equiv (0, -1) \Rightarrow$ The pair of tangents from T is given by $S_1^2 = SS_{11} \Rightarrow (-y - x)^2 = (x^2 + y^2 - 2x)\,1 \Rightarrow x(y + 1) = 0$

The sides of the triangle are $x + y = 1$,

$x = 0$, $y + 1 = 0 \Rightarrow$ Area $= \frac{1}{2}\cdot2\cdot2 = 2$.

10. (b) : (P) $\to$ (1), (Q) $\to$ (2), (R) $\to$ (4), (S) $\to$(3)

(P) $x^2 + y^2 = 2cy$. Differentiating, $x + yy_1 = cy_1$.

Eliminating c, $2y(x + yy_1) = y_1(x^2 + y^2)$

$\Rightarrow$ $(x^2 - y^2)y_1 = 2xy$

(Q) $x^2 + y^2 = 2cx$. Differentiating, $x + yy_1 = c$

Eliminating c, $2x(x + yy_1) = x^2 + y^2$

$\Rightarrow$ $2xyy_1 = y^2 - x^2$

(R) $(x - h)^2 + y^2 = 1$. Differentiating, $x - h + yy_1 = 0$.

Eliminating $x - h$, $1 - y^2 = y^2y_1^2 \Rightarrow (1 + y_1^2)y^2 = 1$

(S) $x^2 + (y - k)^2 = 1$. Differentiating, $x + (y - k)y_1 = 0$.

Eliminating $y - k$, $y_1^2(1 - x^2) = x^2$

MATH archives

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE Main & Advanced Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE Main & Advanced. In every issue of MT, challenging problems are offered with detailed solution. The reader's comments and suggestions regarding the problems and solutions offered are always welcome.

Directions (1-4) : In the following questions, a statement of assertion (Statement-1) is followed by a statement of reason (Statement-2). Mark the correct choice as :

(a) If both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.

(b) If both Statement-1 and Statement-2 are true but Statement-2 is not the correct explanation of Statement-1.

(c) If Statement-1 is true but Statement-2 is false.

(d) If Statement-1 is false and Statement-2 is true.

1. **Statement-1 :** If $a > b > c$ and $a^3 + b^3 + c^3 = 3abc$, then the quadratic equation $ax^2 + bx + c = 0$ has roots of opposite sign.

Statement-2 : If roots of a quadratic equation $ax^2 + bx + c = 0$ are of opposite sign, then product of roots is negative and $|\text{sum of roots}| \geq 0$.

2. **Statement-1 :** A relation R on the set of complex numbers defined by $Z_1RZ_2 \Leftrightarrow \dfrac{Z_1 - Z_2}{Z_1 + Z_2}$ is real, then R is an equivalence relation.

Statement-2 : The 'R' relation is reflexive and symmetric but not transitive.

3. **Statement-1 :** If A and B are two events such that $0 < P(A), P(B) < 1$, then $P(A/\overline{B}) + P(\overline{A}/\overline{B}) = 3/2$

Statement-2 : If A and B are two events such that $0 < P(A), P(B) < 1$, then $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ and $P(\overline{B}) = P(A \cap \overline{B}) + P(\overline{A} \cap \overline{B})$

4. **Statement-1:** If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then equation $\vec{r} \times (2\hat{i} - \hat{j} + 3\hat{k}) = 3\hat{i} + \hat{k}$ represents a straight line.

Statement-2: If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then equation $\vec{r} \times (\hat{i} + 2\hat{j} - 3\hat{k}) = 2\hat{i} - \hat{j}$ represents a straight line.

5. If the pairs of lines $x^2 + 2xy + ay^2 = 0$ and $ax^2 + 2xy + y^2 = 0$ have exactly one line in common then the joint equation of the other two lines is given by

(a) $3x^2 + 8xy - 3y^2 = 0$ (b) $3x^2 + 10xy + 3y^2 = 0$

(c) $y^2 + 2xy - 3x^2 = 0$ (d) $x^2 + 2xy - 3y^2 = 0$

6. If $\cos x = \tan y$, $\cos y = \tan z$, $\cos z = \tan x$, then the value of $\sin x$ is

(a) $2\cos 180°$ (b) $\cos 18°$

(c) $\sin 18°$ (d) $2\sin 18°$

7. The distance of the point $(1,0,-3)$ from the plane $x - y - z = 9$ measured parallel to the line; $\dfrac{x-2}{2} = \dfrac{y+2}{3} = \dfrac{z-6}{6}$ is

(a) 6 units (b) 5 units

(c) 8 units (d) 7 units

8. If $(a\sec\theta, b\tan\theta)$ and $(a\sec\phi, b\tan\phi)$ are the ends of a focal chord of $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, then $\tan\dfrac{\theta}{2}\tan\dfrac{\phi}{2}$ equals to

(a) $\dfrac{e-1}{e+1}$ (b) $\dfrac{1-e}{1+e}$

(c) $\dfrac{1+e}{1-e}$ (d) $\dfrac{e+1}{e-1}$

By : **Prof. Shyam Bhushan,** Director, Narayana IIT Academy, Jamshedpur. Mob. : 09334870021

9. A real-valued function $f(x)$ satisfies the functional equation $f(x-y)=f(x)f(y)-f(a-x)f(a+y)$, where a is a given constant and $f(0)=1$. Then $f(2a-x)$ is equal to

(a) $f(x)$ (b) $-f(x)$

(c) $f(-x)$ (d) $f(a)+f(a-x)$

10. Let $f(x)=\int \frac{x^2dx}{(1+x^2)(1+\sqrt{1+x^2})}$ and $f(0)=0$, then the value of $f(1)$ is

(a) $\log(1+\sqrt{2})$ (b) $\log(1+\sqrt{2})-\frac{\pi}{4}$

(c) $\log(1+\sqrt{2})+\frac{\pi}{2}$ (d) $\log(\sqrt{2}-1)-\frac{\pi}{4}$

SOLUTIONS

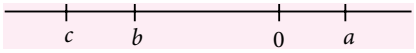
1. (a) : $a>b>c \Rightarrow a, b, c$ are distinct real

Also, $a^3+b^3+c^3-3abc=0$

$$\Rightarrow \left(\frac{a+b+c}{2}\right)[(a-b)^2+(b-c)^2+(c-a)^2]=0$$

As a,b,c are distinct $\therefore \quad a+b+c=0$.

Hence, $x=1$ is a root of $ax^2+bx+c=0$.



$a+b+c=0$ and $a>b>c \Rightarrow a$ and c are of opposite sign otherwise $a+b+c\neq 0$, therefore c/a is negative

2. (c) : $Z_1 R Z_1 \Rightarrow \frac{Z_1-Z_1}{Z_1+Z_1}=0$

$\therefore$ R is reflexive because 0 is real.

$$Z_1RZ_2 \Rightarrow \frac{Z_1-Z_2}{Z_1+Z_2} \text{ is real} \Rightarrow -\left(\frac{Z_2-Z_1}{Z_1+Z_2}\right) \text{is real}$$

$\therefore$ R is symmetric.

For transitivity,

Let $Z_1=a_1+ib_1$, $Z_2=a_2+ib_2$ and $Z_3=a_3+ib_3$

$$Z_1RZ_2 \Rightarrow \frac{Z_1-Z_2}{Z_1+Z_2} \text{ is real}$$

$\Rightarrow (a_1+a_2)(b_1-b_2)-(a_1-a_2)(b_1+b_2)=0$

$$\therefore \quad 2a_2b_1-2b_2a_1=0 \Rightarrow \frac{a_1}{b_1}=\frac{a_2}{b_2}$$

$$\text{Similarly, } Z_1RZ_3 \Rightarrow \frac{a_2}{b_2}=\frac{a_3}{b_3}$$

$\therefore$ $Z_2RZ_3 \Rightarrow R$ is transitive.

3. (d) : Since, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$ (by definition)

$P(\overline{B})=P(A\cup\overline{A})\cap\overline{B}=P(A\cap\overline{B})\cup(\overline{A}\cap\overline{B})$

$\therefore$ Statement-2 is true.

Now, $P(A/\overline{B})+P(\overline{A}/\overline{B})=\frac{P(A\cap\overline{B})}{P(\overline{B})}+\frac{P(\overline{A}\cap\overline{B})}{P(\overline{B})}$

$$\frac{P(A\cap\overline{B})+P(\overline{A}\cap\overline{B})}{P(\overline{B})}=\frac{P(\overline{B})}{P(\overline{B})}=1$$

Therefore, Statement-1 is false

4. (d) : Statement-2 : $\vec{r}\times(\hat{i}+2\hat{j}-3\hat{k})$

$=\hat{i}(-3y-2z)-\hat{j}(-3x-z)+\hat{k}(2x-y)$

$\therefore$ $-3y-2z=2$, $3x+z=-1$, $2x-y=0$

i.e., $-6x-2z=2$, $3x+z=-1$

$\therefore$ Straight line, $2x-y=0$, $3x+z=-1$

Statement-1 : $\vec{r}\times(2\hat{i}-\hat{j}+3\hat{k})$

$=\hat{i}(3y+z)-\hat{j}(3x-2z)+\hat{k}(-x-2y)$

$\therefore$ $3y+z=3$, $3x-2z=0$, $-x-2y=1$

i.e., $3x-2(3-3y)=0$

$\Rightarrow 3x+6y=6 \Rightarrow x+2y=2$

Now, $x+2y=-1$, $x+2y=2$ are parallel planes

Therefore, $\vec{r}\times(2\hat{i}-\hat{j}+3\hat{k})=3\hat{i}+\hat{k}$ is not a straight line.

5. (b) : Let $y=mx$ be a line common to the given pairs of lines, then $am^2+2m+1=0$ and $m^2+2m+a=0$

$$\Rightarrow \quad \frac{m^2}{2(1-a)}=\frac{m}{a^2-1}=\frac{1}{2(1-a)}$$

$$\Rightarrow \quad m^2=1 \text{ and } m=-\frac{a+1}{2}$$

$\Rightarrow (a+1)^2=4 \Rightarrow a=1$ or -3

But for $a=1$, the two pairs have both the lines common, so $a=-3$

Now, $x^2+2xy+ay^2=x^2+2xy-3y^2=(x-y)(x+3y)$

and $ax^2+2xy+y^2=-3x^2+2xy+y^2=-(x-y)(3x+y)$

$\therefore$ Slope m of the line common to both the pairs is 1

So joint equation of the required lines is

$(x+3y)(3x+y)=0$

$\Rightarrow 3x^2+10xy+3y^2=0$

6. (d) : We have, $\cos x=\tan y$

$\cos^2x=\sec^2y-1$

$\Rightarrow \cos^2x=\cot^2z-1$

[$\because \cos y=\tan z$, $\therefore \sec y=\cot z$]

$\Rightarrow 1+\cos^2x=\cot^2z$

$$\Rightarrow \quad 1+\cos^2x=\frac{\sin^2x}{\cos^2x-\sin^2x}$$

$$\therefore \quad \sin^2x=\left(\frac{\sqrt{5}-1}{2}\right)^2 \Rightarrow \sin x=\frac{\sqrt{5}-1}{2}=2\sin 18^\circ$$

Contd. on Page no. 73

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Consider the function

$$f(x) = \min\left\{|x|, \sqrt{1-x^2}\right\}, -1 \le x \le 1$$

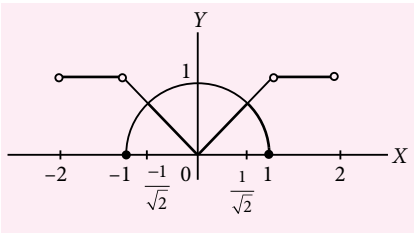
$$= [|x|],\ 1 \le |x| \le 2$$

Plot the curve and discuss its continuity and differentiability from the graph.

(Ritu Jain, Gujarat)

Ans. The given figure shows the curves

$y_1 = |x|, \quad -1 \le x \le 1$



$y_2 = \sqrt{1-x^2}$, $-1 \le x \le 1$

which is the upper half of the circle $x^2 + y^2 = 1$

and $y_3 = [|x|], x \in (-2, -1) \cup (1, 2)$

The portion shown in thicker strokes, represents the curve

$$y = \min\left(|x|, \sqrt{1-x^2}\right), -1 \le x \le 1$$

$$= [|x|],\ 1 < |x| < 2$$

From the graph, we can see that $f(x)$ is continuous everywhere on $(-2, 2) - \{-1, 1\}$

and differentiable everywhere on $(-2, 2) - \left\{\pm 1, \frac{\pm 1}{\sqrt{2}}, 0\right\}$.

2. Evaluate the indefinite integral :

$$\int \frac{1}{\sin^6 x + \cos^6 x}\,dx$$

(Priyansh Sharma, Delhi)

Ans. Let $I = \int \frac{dx}{\sin^6 x + \cos^6 x} = \int \frac{\sec^6 x}{1+\tan^6 x}\,dx$

$$= \int \frac{(1+\tan^2 x)^2}{1+\tan^6 x}\sec^2 x\,dx$$

Putting $\tan x = t$ and $\sec^2 x\,dx = dt$, we have

$$I = \int \frac{(1+t^2)^2}{1+t^6}\,dt = \int \frac{1+t^2}{t^4 - t^2 + 1}\,dt$$

$$= \int \frac{1+\frac{1}{t^2}}{t^2 + \frac{1}{t^2} - 1}\,dt = \int \frac{d\left(t - \frac{1}{t}\right)}{\left(t-\frac{1}{t}\right)^2 + 1}$$

$$= \tan^{-1}\left(t - \frac{1}{t}\right) + C = \tan^{-1}(\tan x - \cot x) + C.$$

3. Suppose m people are asked a question successively in a random order and only 2 of them know the answer.

(i) If $m > 5$, find the probability that the first three of those asked do not know the answer.

(ii) Find the probability that the r^{th} person asked is the first to know the answer.

(Rajnish, Jharkhand)

Ans. (i) The first 3 persons will not answer if they are chosen out of the $(m - 2)$ persons who do not know the answer which can be done in ${}^{m-2}C_3$ ways.

The total number of ways in which the first 3 persons can be chosen, is ${}^{m}C_3$.

$\therefore$ Required probability $= \frac{{}^{m-2}C_3}{{}^{m}C_3}$.

(ii) The first person to know the answer will be the r^{th} person, if the first $(r - 1)$ persons are chosen from the $(m - 2)$ persons who do not know the answer and the r^{th} person is chosen from the 3 persons who know the answer.

Let E = {first $(r - 1)$ persons do not know the answer}

and F = {r^{th} person knows the answer}.

Hence, required probability is

$$P(E \cap F) = P(E)\,P(F/E) = \frac{{}^{m-2}C_{r-1}}{{}^{m}C_{r-1}} \cdot \frac{3}{m-r+1}.$$

CLASS XI | Series 2

CBSE DRILL

Synopsis and Chapter wise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2018-19.

Relations & Functions

ORDERED PAIR

Let A and B be two non-empty sets. If $a \in A$ and $b \in B$, then an element of the form (a, b) is called an ordered pair.

CARTESIAN PRODUCT OF SETS

Cartesian product of two sets A and B is denoted and defined as, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

Note :

(i) Cartesian product of two sets is not commutative in general.

(ii) $(a, b) = (p, q) \Leftrightarrow a = p$ and $b = q$

(iii) If $n(A) = p$, $n(B) = q$, then $n(A \times B) = pq$

RELATIONS

A relation R, from a non-empty set A to another non-empty set B, is a subset of $A \times B$ which is derived by describing a relationship between the first and second element of ordered pairs in $A \times B$. The second element is called image of the first element.

Domain, Range and Co-domain of a Relation

- **Domain :** For any relation R from A to B, the domain is the set of all those elements $a \in A$ s.t. $(a, b) \in R$ for some $b \in B$.
- **Range :** For any relation R from A to B, the range is the set of all those elements $b \in B$ s.t. $(a, b) \in R$ for some $a \in A$.
- **Co-domain :** For any relation R from A to B, B is called the co-domain of R.
- The whole Set B is called the co-domain of R.

Note: Range $\subseteq$ Co-domain

FUNCTIONS

A relation f from set A to set B is said to be a function, if each element of A has image in B.
No element of A has more than one image in B.

Domain, Range and Co-domain of a Function

Let $f : A \to B$ be a function, then

- Set A is the domain of f.
- The set of images of all elements of A is the range of f.
- Set B is the co-domain of f.

Types of Functions

- **Identity Function :** The function $f : R \to R$ defined by $y = f(x) = x$ for each $x \in R$.
  Domain of $f = R$. Range of $f = R$.
- **Constant Function :** The function $f : R \to R$ defined by $y = f(x) = c$, $x \in R$, where c is a constant $\in R$.
  Domain of $f = R$.
  Range of $f = \{c\}$.

- **Polynomial Function :** A real valued function $f: R \to R$ defined by $y = f(x) = a_0 + a_1x + ... + a_nx^n$, where $n \in N$ and $a_0, a_1, a_2, ..., a_n \in R$ for each $x \in R$.
- **Rational Function :** These are the real functions of the type $\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial functions of x defined in a domain, where $g(x) \neq 0$.
- **Modulus Function :** The real function $f : R \to R$ defined by

$$f(x) = |x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases} \forall x \in R.$$

Domain of $f = R$. Range of $f = R^+ \cup \{0\}$.
- **Greatest Integer Function :** The real function $f : R \to R$ defined by $f(x) = [x]$, $x \in R$ assumes the value of the greatest integer, less than or equal to x. Domain of $f = R$, Range of $f = Z$.
- **Signum Function :** The real function $f : R \to R$ defined by

$$f(x) = \begin{cases} \frac{|x|}{x}, x \neq 0 \\ 0, \ x = 0 \end{cases} = \begin{cases} 1, \text{ if } x > 0 \\ 0, \text{if } x = 0 \\ -1, \text{if } x < 0 \end{cases}$$

Domain of $f = R$. Range of $f = \{-1, 0, 1\}$.

ALGEBRA OF REAL FUNCTIONS

Let $f: X \to R$ and $g: X \to R$ be two real functions, then

- **Addition :** $(f + g)(x) = f(x) + g(x) \ \forall \ x \in X$
- **Subtraction :** $(f - g)(x) = f(x) - g(x) \ \forall \ x \in X$
- **Multiplication :** $(fg)(x) = f(x) \cdot g(x) \ \forall \ x \in X$
- **Division :** $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0, x \in X$
- **Multiplication by a scalar :** Let α be any scalar, then $(\alpha f)(x) = \alpha f(x), \forall \ x \in X$

WORK IT OUT

VERY SHORT ANSWER TYPE

1. Let $A = \{1, 2, 3\}$ and $B = \{4, 5\}$. Find $A \times B$, $B \times A$ and show that $A \times B \neq B \times A$.
2. If $f(x) = x + \frac{1}{x}$, prove that $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$.
3. Let N be the set of all natural numbers and $R = \{(x, y) : x + 2y = 0; y \in N\}$. Is R a relation on N? Give reasons.
4. Let $f: R \to R$ is defined by : $f(x) = \cos x, \forall \ x \in R$. Is f a function on R? If yes, then find its range.
5. If $f(x) = \frac{2\tan x}{1 + \tan^2 x}$, then find $f\left(\frac{\pi}{4}\right)$.

SHORT ANSWER TYPE

6. Let $f: R \to R$ be defined by $f(x) = \sin x$ and $g: R \to R$ be defined by $g(x) = x$, find $f + g, f - g, f \cdot g, \frac{f}{g}$ and $\alpha f, \alpha \in R$.
7. Determine domain of the following relation.
$\{(x, y) : y = |x - 1|, x \in Z \text{ and } |x| \leq 3\}$
8. Find the domain of the function $f(x)$ defined by $f(x) = \sqrt{4 - x} + \frac{1}{\sqrt{x^2 - 1}}$.
9. Find the domain of the function $f(x)$ given by $f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$.
10. Let R be the relation on the set N of natural numbers defined by $R = \{(a, b) : a + 3b = 12, a \in N, b \in N\}$. Find: **(i)** Domain of R **(ii)** Range of R

LONG ANSWER TYPE - I

11. The cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$.
12. Find the domain and range of the function defined by $f(x) = \frac{1}{1 - x^2}$.
13. If for non-zero x, $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \neq b$, then find $f(x)$.
14. If $f(x) = \frac{x - 1}{x + 1}$, $x \neq -1$, then show that $f(f(x)) = -\frac{1}{x}$, provided that $x \neq 0$.
15. If $A = \{x : x \in Z, x < 3\}$, $B = \{x : x \in N, 2 \leq x < 4\}$ and $C = \{3, 4\}$, then verify that $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

LONG ANSWER TYPE - II

16. Find the domain and the range of the function f defined by $f(x) = \frac{x + 2}{|x + 2|}$.
17. Let f, g be two functions defined by $f(x) = \sqrt{x + 1}$ and $g(x) = \sqrt{9 - x^2}$, then find
(i) $\frac{f}{g}$ **(ii)** $\frac{g}{f}$ **(iii)** $2f - \sqrt{5}g$
(iv) $f^2 + 7f$ **(v)** $\frac{3}{g}$.

18. Find the domain and range of the function
$f(x)=\dfrac{x^2-x+1}{x^2+x+1}$.

19. Given $A=\{1,2,3,4,5\}$ and $R=\{(x,y):x\in A, y\in A\}$. Find the set of ordered pairs which satisfy the conditions given below :
(i) $x+y=5$ **(ii)** $x+y<5$
(iii) $x+y>8$.

20. The relations f and g are defined by
$$f(x)=\begin{cases}x^2, & 0\le x\le 3\\ 3x, & 3\le x\le 10\end{cases} \text{ and } g(x)=\begin{cases}x^2, & 0\le x\le 2\\ 3x, & 2\le x\le 10\end{cases}.$$
Show that f is a function and g is not a function.

SOLUTIONS

1. Given, $A=\{1,2,3\}$ and $B=\{4,5\}$
$A\times B=\{(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)\}$
$B\times A=\{(4,1),(4,2),(4,3),(5,1),(5,2),(5,3)\}$
Hence, $A\times B\neq B\times A$.

2. We have, $f(x)=x+1/x$
$$\therefore\ f(x^3)=x^3+\frac{1}{x^3} \text{ and } [f(x)]^3=\left(x+\frac{1}{x}\right)^3$$
$$\text{Now, } [f(x)]^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=f(x^3)+3f(x)$$
$$\text{Also, } f(x)=f\left(\frac{1}{x}\right)$$
$$\therefore\ [f(x)]^3=f(x^3)+3f\left(\frac{1}{x}\right)$$

3. We have, $x+2y=0\Rightarrow x=-2y; y\in N$
Then $x=-2,-4,-6,\ldots$
Clearly, $(x,y)=(-2,1)$ for $y=1$
So, $(-2,1)\in R$ whereas $(-2,1)\notin N\times N$
$\therefore\ R\not\subseteq N\times N$. Hence R is not a relation on N.

4. Since for each $x\in R$, $\cos x$ is unique real number, therefore, f is a function.
Since, $-1\le\cos x\le 1$, so range of $f=[-1,1]$.

5. $f(x)=\dfrac{2\tan x}{1+\tan^2 x}\ \therefore\ f\left(\dfrac{\pi}{4}\right)=\dfrac{2\tan\frac{\pi}{4}}{1+\tan^2\frac{\pi}{4}}=\dfrac{2\cdot 1}{1+1}=\dfrac{2}{2}=1.$

6. Given, $f(x)=\sin x$ and $g(x)=x$
Now, $f+g:R\to R,\ (f+g)(x)=f(x)+g(x)=\sin x+x$
$(f-g):R\to R,\ (f-g)(x)=f(x)-g(x)=\sin x-x$
$(f\cdot g):R\to R,\ (f\cdot g)(x)=f(x)\cdot g(x)=(\sin x)x=x\sin x$
$$\frac{f}{g}:R\to R,\ \left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{\sin x}{x},\ x\neq 0$$
$\alpha f:R\to R,\ (\alpha f)(x)=\alpha f(x)=\alpha\sin x$

7. Let $R=\{(x,y):y=|x-1|, x\in Z \text{ and } |x|\le 3\}$
Then, $R=\{(x,y):y=|x-1|, x\in Z \text{ and } -3\le x\le 3\}$
$[\because |x|\le 3\Leftrightarrow -3\le x\le 3]$
$=\{(x,y):y=|x-1|, x=-3,-2,-1,0,1,2,3\}$
$\therefore\ R=\{(-3,4),(-2,3),(-1,2),(0,1),(1,0),(2,1),(3,2)\}$
$\therefore$ Domain of R = Set of first components of all ordered pairs $=\{-3,-2,-1,0,1,2,3\}$

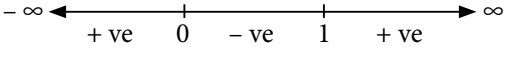
8. Clearly, $f(x)$ is defined for $4-x\ge 0$ and $x^2-1>0$
$\Rightarrow\ x-4\le 0$ and $(x-1)(x+1)>0$
$\Rightarrow\ x\le 4$ and $(x<-1 \text{ or } x>1)$
$\Rightarrow\ x\in(-\infty,-1)\cup(1,4]$.
Hence, Domain of $f(x)=(-\infty,-1)\cup(1,4]$.

9. Given, $f(x)=\dfrac{1}{\sqrt{[x]^2-[x]-6}}$.
For D_f, $f(x)$ must be a real number
$\Rightarrow\ [x]^2-[x]-6>0\Rightarrow([x]+2)([x]-3)>0$
$\Rightarrow\ [x]<-2 \text{ or } [x]>3\Rightarrow x<-2, \text{ or } x\ge 4$
$\Rightarrow\ D_f=(-\infty,-2)\cup[4,\infty)$.

10. We have, $R=\{(a,b):a+3b=12\ a,b\in N\}$
$\therefore\ a=12-3b$
Putting $b=1,2,3$ we get $a=9,6,3$ respectively.
For $b=4$, we get $a=0\notin N$. Also, for $b>4$, $a\notin N$.
$\therefore\ R=\{(9,1),(6,2),(3,3)\}$
(i) Domain of $R=\{9,6,3\}$ **(ii)** Range of $R=\{1,2,3\}$

11. Given, $n(A\times A)=9$
$\Rightarrow\ n(A)\cdot n(A)=9\ \ \therefore\ \ n(A)=3$
Now, $(-1,0)\in A\times A\Rightarrow -1\in A$ and $0\in A$
Again, $(0,1)\in A\times A\Rightarrow 0\in A$ and $1\in A$
$\therefore\ -1,0,1\in A$
But A has exactly three elements, $\therefore\ A=\{-1,0,1\}$.
$\therefore$ Remaining elements of $A\times A$ are $\{(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0),(1,1)\}$.

12. Let $y=f(x)$, then $y=\dfrac{1}{1-x^2}$ ...(i)
For y to be real, $1-x^2\neq 0\Rightarrow x^2\neq 1\Rightarrow x\neq\pm 1$
Hence, domain of $f=R-\{-1,1\}$
From (i), $1-x^2=\dfrac{1}{y}\Rightarrow x^2=1-\dfrac{1}{y}=\dfrac{y-1}{y}$
$$\Rightarrow\ x=\pm\sqrt{\frac{y-1}{y}}$$
For x to be real, $\dfrac{y-1}{y}\ge 0$; Sign scheme for $\dfrac{y-1}{y}$ is



$\Rightarrow -\infty<y<0 \text{ or } 1\le y<\infty$
Hence, range of $f=(-\infty,0)\cup[1,\infty)=\{y:y<0 \text{ or } y\ge 1\}$.

13. We have, $a f(x)+b f\left(\frac{1}{x}\right)=\frac{1}{x}-5$ ...(i)

Replacing x by $\frac{1}{x}$ we get,

$a f\left(\frac{1}{x}\right)+b f(x)=x-5$ ...(ii)

Applying (i) + (ii), we obtain

$\left\{f(x)+f\left(\frac{1}{x}\right)\right\}(a+b)=x+\frac{1}{x}-10$

$\Rightarrow \quad f(x)+f\left(\frac{1}{x}\right)=\frac{1}{a+b}\left[x+\frac{1}{x}-10\right]$ ...(iii)

Applying (i) – (ii), we obtain

$f(x)-f\left(\frac{1}{x}\right)=\frac{1}{a-b}\left[\frac{1}{x}-x\right]$. ...(iv)

from (iii) and (iv),

$f(x)=\frac{1}{a^2-b^2}\left\{\frac{a}{x}-bx\right\}-\frac{5}{a+b}$

14. We have, $f(x)=\frac{x-1}{x+1}, x\neq -1$

$\Rightarrow \quad f(f(x))=f\left(\frac{x-1}{x+1}\right) \Rightarrow \quad f(f(x))=\frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}$

$\Rightarrow \quad f(f(x))=\frac{x-1-x-1}{x-1+x+1} \Rightarrow \quad f(f(x))=\frac{-2}{2x}=-\frac{1}{x}$

Also, $-\frac{1}{x}$ is meaningful for $x \neq 0$.

Hence, $f(f(x))=-\frac{1}{x}$, provided that $x \neq 0$.

15. Given, $A=\{x : x \in Z, x<3\}=\{0, 1, 2\}$

$B=\{x : x \in N, 2 \leq x<4\}=\{2, 3\}$ and $C=\{3, 4\}$

$\Rightarrow \quad A \cup B=\{0, 1, 2, 3\}$.

$\therefore \quad (A \cup B) \times C=\{(0, 3), (0, 4), (1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$ ...(i)

Also, $A \times C=\{(0, 3), (0, 4), (1, 3), (1, 4), (2, 3), (2, 4)\}$

Also, $B \times C=\{(2, 3), (2, 4), (3, 3), (3, 4)\}$.

$\therefore \quad (A \times C) \cup (B \times C)=\{(0, 3), (0, 4), (1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$ ...(ii)

From (i) and (ii), we find that

$(A \cup B) \times C=(A \times C) \cup (B \times C)$.

16. Given, $f(x)=\frac{x+2}{|x+2|}$

For D_f, $f(x)$ must be a real number.

$\Rightarrow \frac{x+2}{|x+2|}$ must be a real number.

$\Rightarrow \quad |x+2| \neq 0 \Rightarrow x+2 \neq 0 \Rightarrow x \neq -2$

$\Rightarrow \quad D_f$ = set of all real numbers except -2 *i.e.*, $D_f=R-\{-2\}$

For R_f, let $y=\frac{x+2}{|x+2|}$. Now, two cases arises :

Case I. If $x+2>0$, then $|x+2|=x+2$,

$\therefore \quad y=\frac{x+2}{|x+2|}=1$.

Case II. If $x+2<0$, $|x+2|=-(x+2)$,

$\therefore \quad y=\frac{x+2}{|x+2|}=\frac{x+2}{-(x+2)}=-1$.

$\therefore \quad$ On combining two cases, $R_f=\{-1, 1\}$.

17. Given, $f(x)=\sqrt{x+1}$, $g(x)=\sqrt{9-x^2}$

$\Rightarrow \quad D_f=[-1, \infty)$ and $D_g=[-3, 3]$.

Let $D=D_f \cap D_g=[-1, \infty) \cap [-3, 3]=[-1, 3] \neq \phi$, then

(i) $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{\sqrt{x+1}}{\sqrt{9-x^2}}=\sqrt{\frac{x+1}{9-x^2}}$

with domain D_1 where $D_1=D$ except where $g(x)=0 \Rightarrow D_1=[-1, 3)$.

(ii) $\left(\frac{g}{f}\right)(x)=\frac{g(x)}{f(x)}=\frac{\sqrt{9-x^2}}{\sqrt{x+1}}=\sqrt{\frac{9-x^2}{x+1}}$

with domain D_2 where $D_2=D$ except where $f(x)=0 \Rightarrow D_2=(-1, 3]$.

(iii) $(2f-\sqrt{5}g)(x)=2f(x)-\sqrt{5}g(x)$

$=2\sqrt{x+1}-\sqrt{5}\sqrt{9-x^2}$ with domain $[-1, 3]$.

(iv) $(f^2+7f)(x)=(f(x))^2+7f(x)=x+1+7\sqrt{x+1}$ with domain $[-1, \infty)$.

(v) $\left(\frac{3}{g}\right)(x)=\frac{3}{g(x)}=\frac{3}{\sqrt{9-x^2}}$ with domain D_3 where $D_3=D_g$ except where $g(x)=0 \Rightarrow D_3=(-3, 3)$.

18. Given, $f(x)=\frac{x^2-x+1}{x^2+x+1}$.

For D_f, $f(x)$ must be real number.

$\Rightarrow \frac{x^2-x+1}{x^2+x+1}$ must be a real number

$\Rightarrow \quad x^2+x+1 \neq 0$.

Now, $x^2+x+1=0 \Rightarrow x=\frac{-1 \pm \sqrt{-3}}{2} \notin R$

$\Rightarrow \quad x^2+x+1 \neq 0$ for all $x \in R \Rightarrow D_f=R$.

For R_f, let $y=\frac{x^2-x+1}{x^2+x+1}$ ...(i)

As $0 \in D_f$, on putting $x=0$ in (i), we get $y=1$ $\Rightarrow 1 \in R_f$.

When $y \neq 1$, from (i), we get $(x^2+x+1)y=x^2-x+1$

$\Rightarrow \quad (y-1)x^2+(y+1)x+(y-1)=0 \qquad (\because x \in R)$

which is a quadratic in x having real roots.

$\Rightarrow$ discriminant $\geq 0 \Rightarrow (y+1)^2 - 4(y-1)(y-1) \geq 0$

$\Rightarrow$ $y^2 + 2y + 1 - 4y^2 + 8y - 4 \geq 0$

$\Rightarrow$ $-3y^2 + 10y - 3 \geq 0 \Rightarrow y^2 - \frac{10}{3}y + 1 \leq 0$

$\Rightarrow$ $\left(y - \frac{1}{3}\right)(y-3) \leq 0$

$\Rightarrow$ $\frac{1}{3} \leq y \leq 3, y \neq 1$ but $1 \in R_f \Rightarrow R_f = \left[\frac{1}{3}, 3\right]$.

19. Given, $A = \{1,2,3,4,5\}$ and $R = \{(x,y) : x \in A, y \in A\}$.
i.e., $R = A \times A$.
$R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2), (4,3), (4,4), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5)\}$.
Thus, R has 25 different ordered pairs.

(i) The ordered pairs of R which satisfy the condition $x + y = 5$ are $\{(1,4), (2,3), (3,2), (4,1)\}$.

(ii) The ordered pairs of R which satisfy the condition $x + y < 5$ are
$\{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\}$.

(iii) The ordered pairs of R which satisfy the condition $x + y > 8$ are $\{(4,5), (5,4), (5,5)\}$.

20. Given $f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$.

Note that $D_f = [0, 10]$.
For $0 \leq x \leq 3$, $f(x) = x^2$, so $f(3) = 3^2 = 9$;
and for $3 \leq x \leq 10$, $f(x) = 3x$, so $f(3) = 3 \times 3 = 9$.
Thus, for each element of the domain of f has a unique image. Therefore, f is a function.

$$g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}.$$

Note that $D_g = [0, 10]$.
For $0 \leq x \leq 2$, $g(x) = x^2$, so $g(2) = 2^2 = 4$;
and for $2 \leq x \leq 10$, $g(x) = 3x$, so $g(2) = 3 \times 2 = 6$.
Thus, the element 2 of the domain of g has two different images. Therefore, g is not a function.

MATH archives

Contd. from Page no. 67

7. **(d) :** Equation of line passing through $Q(1,0,-3)$ and parallel to given line $\frac{x-1}{2} = \frac{y-0}{3} = \frac{z+3}{-6} = r$

Coordinate of any point on the line is
$(2r+1, 3r, -6r-3)$ (say P)
$\therefore$ $(2r+1) - (3r) - (-6r-3) = 9$
$2r + 1 - 3r + 6r + 3 = 9 \Rightarrow r = 1$
$\therefore$ Coordinate of $P(3, 3, -9)$
$\Rightarrow$ Distance between $Q(1, 0, -3)$ and $P(3, 3, -9)$

$= \sqrt{(3-1)^2 + (3-0)^2 + (-9+3)^2}$

$= \sqrt{4+9+36} = 7$ units.

8. **(b) :** The equation of the chord joining $(a\sec\theta, b\tan\theta)$ and $(a\sec\phi, b\tan\phi)$

$$\frac{x}{a}\cos\left(\frac{\theta-\phi}{2}\right) - \frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right) = \cos\frac{\theta+\phi}{2}$$

This passes through $(ae, 0)$, therefore

$$e\cos\left(\frac{\theta-\phi}{2}\right) = \cos\left(\frac{\theta+\phi}{2}\right)$$

Applying componendo and dividendo, we get

$$\frac{e-1}{e+1} = -\tan\frac{\theta}{2}\tan\frac{\phi}{2} \Rightarrow \tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{1-e}{1+e}$$

9. **(b) :** We have, $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$ ...(i)

Putting $x = a$ and $y = x - a$ in (i), we get
$f(a-(x-a)) = f(a)f(x-a) - f(0)f(x)$ ...(ii)
Putting $x = 0$, $y = 0$ in (i) we get
$f(0) = f(0)f(0) - f(a)f(a)$
$\Rightarrow$ $f(0) = (f(0))^2 - (f(a))^2 \Rightarrow f(a) = 0$
$\therefore$ From (ii), $f(2a - x) = -f(x)$

10. **(b) :** $f(x) = \int \frac{x^2 dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

$$f(x) = \int \frac{x^2 dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$$

$$= \int \frac{\tan^2\theta \sec^2\theta d\theta}{\sec^2\theta(1+\sec\theta)} = \int \frac{\tan^2\theta d\theta}{1+\sec\theta} = \int \frac{\sin^2\theta d\theta}{\cos\theta(1+\cos\theta)}$$

$$= \int \frac{(1-\cos\theta)d\theta}{\cos\theta} = \int \sec\theta d\theta - \int d\theta$$

$$= \log\left(x + \sqrt{1+x^2}\right) - \tan^{-1}x + c$$

Given, $f(0) = 0$
$\Rightarrow$ $0 = \log 1 - 0 + C \Rightarrow C = 0$

Now, $f(1) = \log(1+\sqrt{2}) - \frac{\pi}{4}$

MPP-2 MONTHLY Practice Problems

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Complex Numbers and Quadratic Equations

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. If $z = x + iy$ and $x^2 + y^2 = 16$, then the range of $||x| - |y||$ is :

(a) [0, 4] (b) [0, 2]
(c) [2, 4] (d) None of these

2. The minimum value of $x^2 + 2xy + 3y^2 - 6x - 2y$, where $x, y \in R$, is equal to

(a) –9 (b) –11 (c) –12 (d) –10

3. If $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$, then $z_1^2 + z_2^2 + z_3^2$ equals

(a) –3 (b) $-\frac{1}{3}$
(c) 0 (d) 3

4. If $\frac{3}{2+\cos\theta+i\sin\theta} = a + ib$, then $(a - 2)^2 + b^2$ equals

(a) 0 (b) 1
(c) –1 (d) 2

5. If x_1, x_2 are the roots of $ax^2 + bx + c = 0$ and $x_1 + d$, $x_2 + d$ are the roots of $px^2 + qx + r = 0$, where $d \neq 0$, then :

(a) $b^2 - 4ac = q^2 - 4qr$ (b) $\frac{a}{p+d} = \frac{b}{q+d} = \frac{c}{d+r}$
(c) $d = \frac{bp-aq}{2ap}$ (d) $a + d = p$

6. If 1, ω, ω^2, ..., ω^{n-1} are n, n^{th} roots of unity, the value of $(9 - \omega)(9 - \omega^2)$ .... $(9 - \omega^{n-1})$ will be

(a) n (b) 0
(c) $\frac{9^n-1}{8}$ (d) $\frac{9^n+1}{8}$

One or More Than One Option(s) Correct Type

7. If $ax + by = 1$, $cx^2 + dy^2 = 1$ have only one solution, then

(a) $\frac{a^2}{c} + \frac{b^2}{d} = 1$ (b) $x = \frac{a}{c}$
(c) $y = \frac{b}{d}$ (d) None of these

8. The equation $x^{[\log_3 x)^2 - (9/2)\log_3 x + 5]} = 3\sqrt{3}$ has

(a) at least one real solution
(b) exactly three real solutions
(c) exactly one irrational solution
(d) complex roots

9. If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, then :

(a) $\frac{x}{y} + \frac{y}{x} = 2\cos(\theta - \phi)$
(b) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\theta + n\phi)$
(c) $\frac{x^m}{y^n} + \frac{y^n}{x^m} = 2\cos(m\theta - n\phi)$
(d) $xy + \frac{1}{xy} = 2\cos(\theta + \phi)3$

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10. If $\omega \neq 1$ is a cube root of unity, then sum of the series, $S = 1 + 2\omega + 3\omega^2 + ... + 3n\,\omega^{3n-1}$ $(n \in N)$ is

(a) $\dfrac{3n}{\omega - 1}$ (b) $n(\omega^2 - 1)$

(c) 0 (d) None of these

11. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and Re $(z_1\bar{z}_2) = 0$, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies

(a) $|w_1| = 1$ (b) $|w_2| = 1$

(c) $\text{Re}(w_1\bar{w}_2) = 0$ (d) None of these

12. If $x \in R$, then range of $\dfrac{x^2 - 2x + 3}{x^2 - 2x - 8}$ is

(a) $\left[\dfrac{-2}{9}, 1\right)$ (b) $\left[\dfrac{-2}{9}, 1\right]$

(c) $\left(-\infty, -\dfrac{2}{9}\right] \cup (1, \infty)$ (d) None of these

13. If $z_0 = \dfrac{1+i}{2}$, then the value of the product $(1+z_0)(1+z_0^2)(1+z_0^{2^2}).....(1+z_0^{2^n})$ must be

(a) $2^{2^{n+1}}$

(b) $(1+i)\left(1 - \dfrac{1}{2^{2^n}}\right)$, if $n > 1$

(c) $\dfrac{5}{4}(1+i)$, if $n = 1$

(d) 0

Comprehension Type

Modulus of a complex number $x + iy$ is denoted by $|x + iy|$ and is a non-negative real number. Let $z = x + iy$ be the given complex number, then $x - iy$ is called conjugate of z and is denote by $\bar{z}$.

14. Total number of complex numbers z satisfying Re $f(z^2) = 0$, $|z| = \sqrt{3}$ is equal to

(a) 2 (b) 4 (c) 6 (d) 8

15. If $\left|z - \dfrac{1}{z}\right| = 1$, then

(a) $|z|_{max} = \dfrac{\sqrt{5}+1}{2}$ (b) $|z|_{min} = \dfrac{\sqrt{5}+1}{2}$

(c) $|z|_{max} = \dfrac{\sqrt{5}-1}{2}$ (d) $|z|_{min} = \dfrac{\sqrt{5}-1}{4}$

Matrix Match Type

16. Match the following:

	Column I		Column II
P.	If $(3a - 2ib)(2 + i)^2 = 10(1 + i)$, then reciprocal of $(a - 3b)$ is	1.	3
Q.	If $z + z^3 = 0$, then Re(z) is equal to	2.	2
R.	If α is a complex constant such that $\alpha z^2 + z + \bar{\alpha} = 0$ has a real root, then value of $4\lvert\alpha + \bar{\alpha}\rvert$ is	3.	4
S.	If $x, y \in R$, and $2x^2 + 6xy + 5y^2 = 1$, then maximum value of y^2 is	4.	0

	P	Q	R	S
(a)	1	4	3	2
(b)	2	1	3	4
(c)	3	1	4	2
(d)	3	4	1	2

Integer Answer Type

17. If $a_1, a_2, ..., a_n$ are real numbers and $\cos\alpha + i\sin\alpha$ is a root of $x^n + a_1 x^{n-1} + a_2 x^{n-2} + ... + a_{n-1}x + a_n = 0$, find the value of $|a_1\cos\alpha + a_2\cos 2\alpha + ... + a_n\cos(n\alpha)|$.

18. The sum of all the integral roots of $(\log_5 x)^2 + \log_{5x}(5/x) = 1$ is

19. If $|z| = \max\{|z - 1|, |z + 1|\}$, then value of $|z + \bar{z}|$ is

20. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is

◈◈

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted .....
No. of questions correct .....
Marks scored in percentage .....

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CLASS XII | Series 2

CBSE DRILL

Synopsis and Chapter wise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2018-19.

Inverse Trigonometric Functions

DOMAIN AND RANGE OF INVERSE TRIGONOMETRIC FUNCTIONS

Functions	Domain	Range (principal value branch)
$y = \sin^{-1}x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1}x$	$(-\infty, \infty)$	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$y = \text{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$

PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

1. • $\sin^{-1}(\sin x) = x, \forall\, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

• $\cos^{-1}(\cos x) = x, \forall\, x \in [0, \pi]$

• $\tan^{-1}(\tan x) = x, \forall\, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

• $\cot^{-1}(\cot x) = x, \forall\, x \in (0, \pi)$

• $\sec^{-1}(\sec x) = x, \forall\, x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

• $\text{cosec}^{-1}(\text{cosec } x) = x, \forall\, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

2. • $\sin(\sin^{-1}x) = x, \forall\, x \in [-1, 1]$

• $\cos(\cos^{-1}x) = x, \forall\, x \in [-1, 1]$

• $\tan(\tan^{-1}x) = x, \forall\, x \in R$

• $\cot(\cot^{-1}x) = x, \forall\, x \in R$

• $\sec(\sec^{-1}x) = x, \forall\, x \in R - (-1, 1)$

• $\text{cosec}(\text{cosec}^{-1}x) = x, \forall\, x \in R - (-1, 1)$

3. • $\sin^{-1}(-x) = -\sin^{-1}x, \forall\, x \in [-1, 1]$

• $\cos^{-1}(-x) = \pi - \cos^{-1}x, \forall\, x \in [-1, 1]$

• $\tan^{-1}(-x) = -\tan^{-1}x, \forall\, x \in R$

• $\cot^{-1}(-x) = \pi - \cot^{-1}x, \forall\, x \in R$

• $\sec^{-1}(-x) = \pi - \sec^{-1}x, \forall\, x \in (-\infty, -1] \cup [1, \infty)$

• $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x, \forall\, x \in (-\infty, -1] \cup [1, \infty)$

4. • $\sin^{-1}(1/x) = \operatorname{cosec}^{-1}x, \forall\, x \in (-\infty, -1]\cup[1, \infty)$

• $\cos^{-1}(1/x) = \sec^{-1}x, \ \forall\, x \in (-\infty, -1]\cup[1, \infty)$

• $\tan^{-1}(1/x) = \begin{cases} \cot^{-1}x & , \quad \text{for } x > 0 \\ -\pi + \cot^{-1}x & , \quad \text{for } x < 0 \end{cases}$

5. • $\sin^{-1}x + \cos^{-1}x = \pi/2, \ \forall\, x \in [-1, 1]$

• $\tan^{-1}x + \cot^{-1}x = \pi/2, \ \forall\, x \in R$

• $\sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2, \ \forall\, x \in (-\infty, -1]\cup[1, \infty)$

6. • $\tan^{-1}x + \tan^{-1}y$

$$= \begin{cases} \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & , \text{ if } x>0,\ y>0,\ xy<1 \\ \pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & , \text{ if } x>0,\ y>0 \text{ and } xy>1 \\ -\pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right), & \text{ if } x<0,\ y<0 \text{ and } xy>1 \end{cases}$$

• $\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & , \text{ if } x>0,\ y<0, xy>-1 \\ \pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & , \text{ if } x>0,\ y<0 \text{ and } xy<-1 \\ -\pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right), & \text{ if } x<0,\ y>0 \text{ and } xy<-1 \end{cases}$

7. • $\sin^{-1}x + \sin^{-1}y = \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\} & , \quad \text{if } -1 \le x,\ y \le 1 \text{ and } x^2+y^2 \le 1 \\ & \qquad \text{or} \\ & \quad \text{if } xy < 0 \text{ and } x^2+y^2 > 1 \\ \pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\} & , \quad \text{if } 0 < x,\ y \le 1 \text{ and } x^2+y^2 > 1 \\ -\pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}, & \quad \text{if } -1 \le x,\ y < 0 \text{ and } x^2+y^2 > 1 \end{cases}$

• $\sin^{-1}x - \sin^{-1}y = \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\} & , \quad \text{if } -1 \le x,\ y \le 1 \text{ and } x^2+y^2 \le 1 \\ & \qquad \text{or} \\ & \quad \text{if } xy > 0 \text{ and } x^2+y^2 > 1 \\ \pi - \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\} & , \quad \text{if } 0 < x \le 1,\ -1 \le y \le 0 \text{ and } x^2+y^2 > 1 \\ -\pi - \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\}, & \quad \text{if } -1 \le x < 0,\ 0 < y \le 1 \text{ and } x^2+y^2 > 1 \end{cases}$

8. • $\cos^{-1}x + \cos^{-1}y = \begin{cases} \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\} & , \text{ if } -1 \le x,\ y \le 1 \text{ and } x+y \ge 0 \\ 2\pi - \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\}, & \text{ if } -1 \le x,\ y \le 1 \text{ and } x+y \le 0 \end{cases}$

• $\cos^{-1}x - \cos^{-1}y = \begin{cases} \cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\} & , \text{ if } -1 \le x,\ y \le 1 \text{ and } x \le y \\ -\cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\}, & \text{ if } -1 \le y \le 0,\ 0 < x \le 1 \text{ and } x \ge y \end{cases}$

9. • $2\sin^{-1}x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}) & , \text{if } -\dfrac{1}{\sqrt{2}} \le x \le \dfrac{1}{\sqrt{2}} \\ \pi - \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } \dfrac{1}{\sqrt{2}} \le x \le 1 \\ -\pi - \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } -1 \le x \le -\dfrac{1}{\sqrt{2}} \end{cases}$

• $3\sin^{-1}x = \begin{cases} \sin^{-1}(3x-4x^3) & , \quad \text{if } -\dfrac{1}{2} \le x \le \dfrac{1}{2} \\ \pi - \sin^{-1}(3x-4x^3) & , \quad \text{if } \dfrac{1}{2} < x \le 1 \\ -\pi - \sin^{-1}(3x-4x^3), & \quad \text{if } -1 \le x < -\dfrac{1}{2} \end{cases}$

10. • $2\cos^{-1}x=\begin{cases}\cos^{-1}(2x^2-1), & \text{if } 0\le x\le 1\\ 2\pi-\cos^{-1}(2x^2-1), & \text{if } -1\le x\le 0\end{cases}$

• $3\cos^{-1}x=\begin{cases}\cos^{-1}(4x^3-3x), & \text{if } \frac{1}{2}\le x\le 1\\ 2\pi-\cos^{-1}(4x^3-3x), & \text{if } -\frac{1}{2}\le x\le\frac{1}{2}\\ 2\pi+\cos^{-1}(4x^3-3x), & \text{if } -1\le x\le-\frac{1}{2}\end{cases}$

11. • $2\tan^{-1}x=\begin{cases}\tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } -1<x<1\\ \pi+\tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } x>1\\ -\pi+\tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } x<-1\end{cases}$

• $3\tan^{-1}x=\begin{cases}\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & \text{if } -\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}\\ \pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & \text{if } x>\frac{1}{\sqrt{3}}\\ -\pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & \text{if } x<-\frac{1}{\sqrt{3}}\end{cases}$

12. • $2\tan^{-1}x=\begin{cases}\sin^{-1}\left(\frac{2x}{1+x^2}\right), & \text{if } -1\le x\le 1\\ \pi-\sin^{-1}\left(\frac{2x}{1+x^2}\right), & \text{if } x>1\\ -\pi-\sin^{-1}\left(\frac{2x}{1+x^2}\right), & \text{if } x<-1\end{cases}$

• $2\tan^{-1}x=\begin{cases}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & \text{if } 0\le x<\infty\\ -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & \text{if } -\infty<x\le 0\end{cases}$

13. • $\sin^{-1}x=\cos^{-1}\left(\sqrt{1-x^2}\right)=\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$=\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)=\text{cosec}^{-1}\left(\frac{1}{x}\right)$

• $\cos^{-1}x=\sin^{-1}\left(\sqrt{1-x^2}\right)=\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

$=\cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)=\sec^{-1}\left(\frac{1}{x}\right)=\text{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

• $\tan^{-1}x=\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

$=\cot^{-1}\left(\frac{1}{x}\right)=\sec^{-1}\left(\sqrt{1+x^2}\right)=\text{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$

14. • If $x_1, x_2, \ldots, x_n \in R$, then $\tan^{-1}x_1+\tan^{-1}x_2+\ldots.+\tan^{-1}x_n=\tan^{-1}\left(\frac{S_1-S_3+S_5-S_7+\ldots}{1-S_2+S_4-S_6+\ldots}\right)$

where S_k = Sum of the products of $x_1, x_2, \ldots, x_n$ taken k at a time.

WORK IT OUT

VERY SHORT ANSWER TYPE

1. Find the principal value of $\cos^{-1}\left\{\sin\left(\cos^{-1}\frac{1}{2}\right)\right\}$.

2. Find the domain of $\text{cosec}^{-1}(3-2x)$.

3. Write $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$ in simplest form.

4. Evaluate: $\sin^{-1}(\sin(-600°))$.

5. If $\tan^{-1}x+\tan^{-1}y=\frac{4\pi}{5}$, then find $\cot^{-1}x+\cot^{-1}y$.

SHORT ANSWER TYPE

6. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, prove that $x^2+y^2+z^2+2xyz=1$.

7. Solve for x : $\tan(\cos^{-1}x)=\sin\left(\cot^{-1}\frac{1}{2}\right)$

8. Evaluate : $\sin\left(2\cot^{-1}\left(-\frac{5}{12}\right)\right)$.

9. Show that : $\sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos(\tan^{-1}2\sqrt{2})=\frac{14}{15}$

10. If the angle C of a triangle is right angle, then prove that $\tan^{-1}\frac{a}{b+c}+\tan^{-1}\frac{b}{c+a}=\frac{\pi}{4}$, where a, b, c are the lengths of the sides of the triangle opposite to the angles A, B, C respectively.

LONG ANSWER TYPE - I

11. Prove that $\tan^{-1}\frac{yz}{xr}+\tan^{-1}\frac{zx}{yr}+\tan^{-1}\frac{xy}{zr}=\frac{\pi}{2}$, where $x, y, z > 0$ such that $x^2+y^2+z^2=r^2$.

12. Prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2.$$

13. Solve for x : $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$

14. Prove that:

$$\tan\frac{1}{2}\left\{\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right\}=\frac{x+y}{1-xy},$$

if $|x| < 1, y > 0$ and $xy < 1$.

15. Sum the following series to infinity :

$$\tan^{-1}\frac{1}{1+1+1^2}+\tan^{-1}\frac{1}{1+2+2^2}+\tan^{-1}\frac{1}{1+3+3^2}+.....$$

LONG ANSWER TYPE - II

16. Show that : $2\tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right]$

$$=\tan^{-1}\left(\frac{\cos\alpha\cos\beta}{\sin\alpha+\sin\beta}\right).$$

17. Solve for x : $\sec^{-1}\frac{x}{a}-\sec^{-1}\frac{x}{b}=\sec^{-1}b-\sec^{-1}a.$

18. Find the greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$.

19. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, then prove that $x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$.

20. Prove that :

$$\frac{\alpha^3}{2}\text{cosec}^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right)+\frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right)$$

$= (\alpha + \beta)(\alpha^2 + \beta^2)$.

SOLUTIONS

1. We know that $\cos^{-1}\frac{1}{2}=\frac{\pi}{3}$.

$$\therefore\quad \cos^{-1}\left\{\sin\left(\cos^{-1}\frac{1}{2}\right)\right\}=\cos^{-1}\left(\sin\frac{\pi}{3}\right)$$

$$=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6} \qquad \left[\because\quad \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}\right]$$

2. For the domain of $\text{cosec}^{-1}(3 - 2x)$, $|3 - 2x| \geq 1$

$\Rightarrow$ $3 - 2x \leq -1$ or $3 - 2x \geq 1$

$\Rightarrow$ $-2x \leq -4$ or $-2x \geq -2$

$\Rightarrow$ $x \geq 2$ or $x \leq 1$.

$\therefore$ Domain of $\text{cosec}^{-1}(3 - 2x)$ is $(-\infty, 1] \cup [2, \infty)$.

3. Put $x = a\tan y \Rightarrow y = \tan^{-1}\frac{x}{a}$.

$$\therefore\quad \tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$$

$$=\tan^{-1}\left(\frac{3a^2\cdot a\tan y-a^3\tan^3 y}{a^3-3a\cdot a^2\tan^2 y}\right)$$

$$=\tan^{-1}\left(\frac{3\tan y-\tan^3 y}{1-3\tan^2 y}\right)$$

$= \tan^{-1}(\tan 3y) = 3y = 3\tan^{-1}\frac{x}{a}$.

4. $\sin^{-1}(\sin(-600°))=\sin^{-1}\left\{\sin\left(-600\times\frac{\pi}{180}\right)\right\}$

$$=\sin^{-1}\left\{\sin\left(-\frac{10\pi}{3}\right)\right\}=\sin^{-1}\left(-\sin\frac{10\pi}{3}\right)$$

$$=\sin^{-1}\left\{-\sin\left(3\pi+\frac{\pi}{3}\right)\right\}=\sin^{-1}\left\{-\left(-\sin\frac{\pi}{3}\right)\right\}$$

$$=\sin^{-1}\left(\sin\frac{\pi}{3}\right)=\frac{\pi}{3}$$

5. We have, $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$

$$\Rightarrow\quad \left(\frac{\pi}{2}-\cot^{-1}x\right)+\left(\frac{\pi}{2}-\cot^{-1}y\right)=\frac{4\pi}{5}$$

$$\left[\because\quad \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right]$$

$$\Rightarrow\quad \pi-(\cot^{-1}x+\cot^{-1}y)=\frac{4\pi}{5}$$

$$\Rightarrow\quad \cot^{-1}x+\cot^{-1}y=\pi-\frac{4\pi}{5}=\frac{\pi}{5}$$

6. Let $\cos^{-1}x = \alpha$, $\cos^{-1}y = \beta$ and $\cos^{-1}z = \gamma$

$\Rightarrow$ $\cos\alpha = x$, $\cos\beta = y$ and $\cos\gamma = z$

From question, $\alpha + \beta + \gamma = \pi$ $\therefore$ $\alpha + \beta = \pi - \gamma$

$\therefore$ $\cos(\alpha + \beta) = \cos(\pi - \gamma)$

$\Rightarrow$ $\cos\alpha\cos\beta - \sin\alpha\sin\beta = -\cos\gamma$

$$\Rightarrow\quad xy-\sqrt{1-x^2}\sqrt{1-y^2}=-z$$

$$\Rightarrow\quad xy+z=\sqrt{1-x^2}\sqrt{1-y^2}$$

Squaring, we get

$x^2y^2 + z^2 + 2xyz = 1 - x^2 - y^2 + x^2y^2$

or $x^2 + y^2 + z^2 + 2xyz = 1$.

7. We have, $\tan(\cos^{-1}x) = \sin\left(\cot^{-1}\frac{1}{2}\right)$

$$\Rightarrow\quad \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right)=\sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$$

$$\Rightarrow\quad \frac{\sqrt{1-x^2}}{x}=\frac{2}{\sqrt{5}}\Rightarrow\frac{1-x^2}{x^2}=\frac{4}{5}$$

$$\Rightarrow\quad 4x^2 = 5 - 5x^2 \Rightarrow 9x^2 = 5 \Rightarrow x=\pm\frac{\sqrt{5}}{3}$$

8. Let $\cot^{-1}\left(-\frac{5}{12}\right)=x, 0<x<\pi$

$\Rightarrow \quad \cot x=-\frac{5}{12}, 0<x<\pi.$

For these values of x, cosec $x>0$.

$\therefore \quad \operatorname{cosec} x=\sqrt{1+\cot^2 x}=\sqrt{1+\left(-\frac{5}{12}\right)^2}=\frac{13}{12}$

$\Rightarrow \quad \sin x=\frac{12}{13}.$

Now, $\cos x=\frac{\cos x}{\sin x}\cdot\sin x=\cot x\cdot\sin x$

$=\left(-\frac{5}{12}\right).\frac{12}{13}=-\frac{5}{13}.$

$\therefore \quad \sin\left(2\cot^{-1}\left(-\frac{5}{12}\right)\right)=\sin 2x=2\sin x\cos x=-\frac{120}{169}.$

9. Let $\tan^{-1}\frac{1}{3}=\alpha\Rightarrow\tan\alpha=\frac{1}{3}.$

$$\sin\left(2\tan^{-1}\frac{1}{3}\right)=\sin 2\alpha=\frac{2\tan\alpha}{1+\tan^2\alpha}=\frac{2\times\frac{1}{3}}{1+\left(\frac{1}{3}\right)^2}=\frac{3}{5}$$

Let $\tan^{-1}2\sqrt{2}=\beta\Rightarrow\tan\beta=2\sqrt{2}.$

$\sec\beta=\sqrt{1+\tan^2\beta}=\sqrt{1+\left(2\sqrt{2}\right)^2}=3\Rightarrow\cos\beta=\frac{1}{3}.$

$\Rightarrow \quad \cos\left(\tan^{-1}2\sqrt{2}\right)=\cos\beta=\frac{1}{3}.$

$\therefore \quad \sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)=\frac{3}{5}+\frac{1}{3}=\frac{14}{15}.$

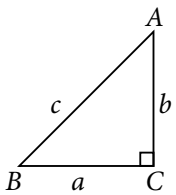
10. In ΔABC, $\angle C=90°$ (given).

$\therefore$ By Pythagoras theorem, $c^2=a^2+b^2$ ...(i)

$$\tan^{-1}\frac{a}{b+c}+\tan^{-1}\frac{b}{c+a}=\tan^{-1}\left(\frac{\frac{a}{b+c}+\frac{b}{c+a}}{1-\frac{a}{b+c}.\frac{b}{c+a}}\right)$$

$$=\tan^{-1}\left(\frac{a(c+a)+b(b+c)}{(b+c)(c+a)-ab}\right)$$

$$=\tan^{-1}\left(\frac{ca+bc+(a^2+b^2)}{bc+ca+c^2}\right)$$



$$=\tan^{-1}\left(\frac{ca+bc+c^2}{bc+ca+c^2}\right)\text{(using (i))}$$

$$=\tan^{-1}(1)=\tan^{-1}\left(\tan\frac{\pi}{4}\right)=\frac{\pi}{4}.$$

11. We have, $x^2+y^2+z^2=r^2$

Also, $\frac{yz}{xr}\times\frac{zx}{yr}=\frac{z^2}{r^2}=\frac{z^2}{x^2+y^2+z^2}<1$

$\therefore \quad \tan^{-1}\frac{yz}{xr}+\tan^{-1}\frac{xz}{yr}+\tan^{-1}\frac{xy}{zr}$

$$=\tan^{-1}\left\{\frac{\frac{yz}{xr}+\frac{xz}{yr}}{1-\frac{yz}{xr}\times\frac{zx}{yr}}\right\}+\tan^{-1}\frac{xy}{zr}$$

$$=\tan^{-1}\left\{\frac{\frac{z(x^2+y^2)}{xyr}}{1-\frac{z^2}{r^2}}\right\}+\tan^{-1}\left(\frac{xy}{zr}\right)$$

$$=\tan^{-1}\left\{\frac{z(x^2+y^2)}{xyr}\times\frac{r^2}{x^2+y^2}\right\}+\tan^{-1}\left(\frac{xy}{zr}\right)$$

$$=\tan^{-1}\left(\frac{zr}{xy}\right)+\tan^{-1}\left(\frac{xy}{zr}\right)$$

$$=\cot^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{xy}{zr}\right)=\frac{\pi}{2}$$

12. Let $x^2=\cos 2y$

$\Rightarrow \quad 2y=\cos^{-1}(x^2), 0\le 2y\le\pi$ *i.e.*, $0\le y\le\frac{\pi}{2}$

$\Rightarrow \quad y=\frac{1}{2}\cos^{-1}(x^2).$

$\therefore \quad \sqrt{1+x^2}=\sqrt{1+\cos 2y}=\sqrt{2\cos^2 y}=\sqrt{2}\cos y$

Also, $\sqrt{1-x^2}=\sqrt{1-\cos 2y}=\sqrt{2\sin^2 y}=\sqrt{2}\sin y.$

$\therefore \quad \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

$$=\tan^{-1}\left(\frac{\sqrt{2}\cos y+\sqrt{2}\sin y}{\sqrt{2}\cos y-\sqrt{2}\sin y}\right)=\tan^{-1}\left(\frac{1+\tan y}{1-\tan y}\right)$$

(Dividing num. and deno. by $\sqrt{2}\ \cos y$)

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+y\right)\right)=\frac{\pi}{4}+y\ =\frac{\pi}{4}+\frac{1}{2}\cos^{-1}(x^2).$$

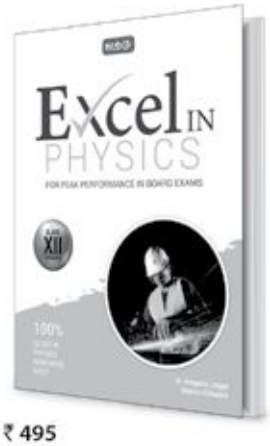
13. We have, $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$

$\Rightarrow \quad \sin^{-1}x+\sin^{-1}(1-x)=\frac{\pi}{2}-\sin^{-1}x$

$\Rightarrow \quad \sin^{-1}(1-x)=\frac{\pi}{2}-2\sin^{-1}x$

$\Rightarrow \quad \sin\left\{\sin^{-1}(1-x)\right\}=\sin\left(\frac{\pi}{2}-2\sin^{-1}x\right)$

Concerned about your performance in Class XII Boards?



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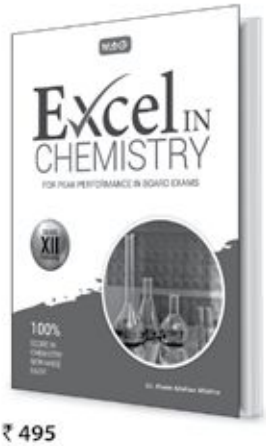

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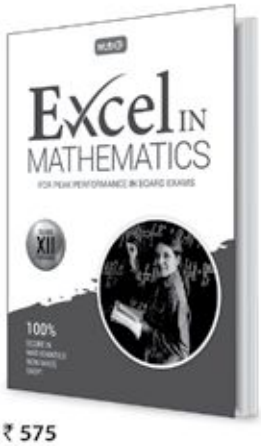

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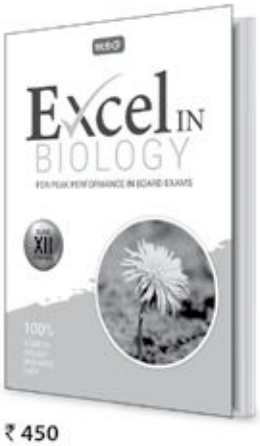

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**Application to read QR codes required*

$\Rightarrow \quad 1 - x = \cos(2\sin^{-1} x)$

$\Rightarrow \quad 1 - x = \cos\{\cos^{-1}(1 - 2x^2)\}$

$[\because 2\sin^{-1} x = \cos^{-1}(1 - 2x^2)$

$\Rightarrow \quad 1 - x = 1 - 2x^2 \Rightarrow 2x^2 - x = 0 \Rightarrow x(2x - 1) = 0$

$\Rightarrow \quad x = 0$ or, $x = \frac{1}{2}$

Clearly, these values satisfy the given equation.

Hence, $x = 0, \frac{1}{2}$ are the roots of the given equation.

14. We know that $\cos^{-1}\frac{1-y^2}{1+y^2} = 2\tan^{-1} y$ for all $y \geq 0$

and $\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1} x$ for all $x \in [-1, 1]$.

$\therefore \quad \text{L.H.S.} = \tan\frac{1}{2}\left\{\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right\}$

$= \tan\frac{1}{2}\left\{2\tan^{-1} x + 2\tan^{-1} y\right\} = \tan(\tan^{-1} x + \tan^{-1} y)$

$= \tan\left\{\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\} \qquad [\because xy < 1]$

$= \frac{x+y}{1-xy} = \text{R.H.S}$

15. Let T_n be the n^{th} term of the given series. Then,

$T_n = \tan^{-1}\left\{\frac{1}{1+n+n^2}\right\} = \tan^{-1}\left\{\frac{(n+1)-n}{1+(n+1)n}\right\}$

$= \tan^{-1}(n + 1) - \tan^{-1} n$

Let S be the sum of the given series to infinity. Then,

$S = \sum_{n=1}^{\infty} T_n \quad \Rightarrow \quad S = \lim_{n\to\infty}\sum_{r=1}^{n} T_r$

$\Rightarrow \quad S = \lim_{n\to\infty}(T_1 + T_2 + \ldots\ldots + T_n)$

$\Rightarrow \quad S = \lim_{n\to\infty}\{(\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2)$

$+ \ldots + (\tan^{-1}(n + 1) - \tan^{-1} n)\}$

$\Rightarrow \quad S = \lim_{n\to\infty}\left\{\tan^{-1}(n+1) - \tan^{-1} 1\right\}$

$= \tan^{-1}\infty - \tan^{-1} 1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

16. As $\frac{1-\sin x}{1+\sin x} = \frac{1-\cos\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)} = \frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}$

$= \tan^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$\Rightarrow \quad \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) = \sqrt{\frac{1-\sin x}{1+\sin x}}.$

$\therefore \quad \text{L.H.S.} = 2\tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right]$

$= 2\tan^{-1}\left(\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}}\sqrt{\frac{1-\sin\beta}{1+\sin\beta}}\right)$

$= \tan^{-1}\left(\frac{2\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}\cdot\frac{1-\sin\beta}{1+\sin\beta}}}{1-\left(\frac{1-\sin\alpha}{1+\sin\alpha}\cdot\frac{1-\sin\beta}{1+\sin\beta}\right)}\right)$

$\left[\because 2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}\right]$

$= \tan^{-1}\left(\frac{2\sqrt{(1-\sin^2\alpha)(1-\sin^2\beta)}}{(1+\sin\alpha)(1+\sin\beta)-(1-\sin\alpha)(1-\sin\beta)}\right)$

$= \tan^{-1}\left(\frac{2\cos\alpha\cos\beta}{2\sin\alpha+2\sin\beta}\right) = \tan^{-1}\left(\frac{\cos\alpha\cos\beta}{\sin\alpha+\sin\beta}\right)$

$= \text{R.H.S.}$

17. Given, $\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1} b - \sec^{-1} a$

$\Rightarrow \quad \cos^{-1}\frac{a}{x} + \cos^{-1}\frac{1}{a} = \cos^{-1}\frac{b}{x} + \cos^{-1}\frac{1}{b}$

$\Rightarrow \quad \cos^{-1}\left(\frac{a}{x}\cdot\frac{1}{a} - \sqrt{1-\left(\frac{a}{x}\right)^2}\sqrt{1-\left(\frac{1}{a}\right)^2}\right)$

$= \cos^{-1}\left(\frac{b}{x}\cdot\frac{1}{b} - \sqrt{1-\left(\frac{b}{x}\right)^2}\sqrt{1-\left(\frac{1}{b}\right)^2}\right)$

$\Rightarrow \quad \frac{1}{x} - \sqrt{1-\frac{a^2}{x^2}}\sqrt{1-\frac{1}{a^2}} = \frac{1}{x} - \sqrt{1-\frac{b^2}{x^2}}\sqrt{1-\frac{1}{b^2}}$

$\Rightarrow \quad \frac{\sqrt{x^2-a^2}\sqrt{a^2-1}}{xa} = \frac{\sqrt{x^2-b^2}\sqrt{b^2-1}}{bx}$

$\Rightarrow \quad b^2(x^2 - a^2)(a^2 - 1) = a^2(x^2 - b^2)(b^2 - 1)$

$\Rightarrow \quad (a^2 b^2 - b^2 - a^2 b^2 + a^2) x^2 = a^2 b^2 (a^2 - 1 - b^2 + 1)$

$\Rightarrow \quad (a^2 - b^2) x^2 = a^2 b^2 (a^2 - b^2) \Rightarrow x^2 = a^2 b^2$

$\Rightarrow \quad x = ab, -ab$ but $x = -ab$ does not satisfy the given equation.

$\therefore \quad x = ab$ is the only solution.

18. We have, $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$

$$= \left\{\left(\sin^{-1}x\right)^2 + \left(\cos^{-1}x\right)^2 + 2\sin^{-1}x\cos^{-1}x\right\}$$

$$- 2\sin^{-1}x\cos^{-1}x$$

$$= (\sin^{-1}x + \cos^{-1}x)^2 - 2\sin^{-1}x\cos^{-1}x$$

$$= \frac{\pi^2}{4} - 2\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$$

$$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

$$= \frac{\pi^2}{4} - \pi\sin^{-1}x + 2\left(\sin^{-1}x\right)^2$$

$$= 2\left\{(\sin^{-1}x)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{8}\right\}$$

$$= 2\left\{\left(\sin^{-1}x\right)^2 - 2\left(\frac{\pi}{4}\right)\sin^{-1}x + \frac{\pi^2}{16} - \frac{\pi^2}{16} + \frac{\pi^2}{8}\right\}$$

$$= 2\left\{\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right\}$$

Now, $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$ for all $x \in [-1,1]$

$$\Rightarrow -\frac{\pi}{2} - \frac{\pi}{4} \le \sin^{-1}x - \frac{\pi}{4} \le \frac{\pi}{2} - \frac{\pi}{4} \text{ for all } x \in [-1,1]$$

$$\Rightarrow -\frac{3\pi}{4} \le \left(\sin^{-1}x - \frac{\pi}{4}\right) \le \frac{\pi}{4} \text{ for all } x \in [-1,1]$$

$$\Rightarrow 0 \le \left(\sin^{-1}x - \frac{\pi}{4}\right)^2 \le \frac{9\pi^2}{16}$$

$$\Rightarrow \frac{\pi^2}{16} \le \left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16} \le \frac{9\pi^2}{16} + \frac{\pi^2}{16}$$

$$\Rightarrow \frac{\pi^2}{16} \le \left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16} \le \frac{5\pi^2}{8}$$

$$\Rightarrow \frac{\pi^2}{8} \le 2\left\{\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right\} \le \frac{5\pi^2}{4}$$

$$\Rightarrow \frac{\pi^2}{8} \le (\sin^{-1}x)^2 + (\cos^{-1}x)^2 \le \frac{5\pi^2}{4}$$

19. Let $\sin^{-1}x = \alpha \Rightarrow x = \sin\alpha$

$\therefore \cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-x^2}$.

Similarly, let $\sin^{-1}y = \beta$ and $\sin^{-1}z = \gamma$, then $\cos\beta = \sqrt{1-y^2}$ and $\cos\gamma = \sqrt{1-z^2}$.

Given, $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi \Rightarrow \alpha + \beta + \gamma = \pi$

$\Rightarrow \alpha + \beta = \pi - \gamma \Rightarrow \cos(\alpha+\beta) = \cos(\pi - \gamma)$

$\Rightarrow \cos\alpha\cos\beta - \sin\alpha\sin\beta = -\cos\gamma$

$\Rightarrow \sqrt{1-x^2}\sqrt{1-y^2} - xy = -\sqrt{1-z^2}$

$\Rightarrow \sqrt{1-x^2}\sqrt{1-y^2} = xy - \sqrt{1-z^2}$

$\Rightarrow (1-x^2)(1-y^2) = \left(xy - \sqrt{1-z^2}\right)^2$

$\Rightarrow 1 - x^2 - y^2 + x^2y^2 = x^2y^2 + (1-z^2) - 2xy\sqrt{1-z^2}$

$\Rightarrow 2xy\sqrt{1-z^2} = x^2 + y^2 - z^2$

$\Rightarrow 4x^2y^2(1-z^2) = (x^2+y^2-z^2)^2$

$\Rightarrow 4x^2y^2 - 4x^2y^2z^2 = x^4 + y^4 + z^4 + 2x^2y^2 - 2x^2z^2 - 2y^2z^2$

$\Rightarrow x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$.

20. We have,

$$\text{L.H.S.} = \frac{\alpha^3}{2}\text{cosec}^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right) + \frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right)$$

$$\Rightarrow \text{L.H.S.} = \frac{\alpha^3}{2\sin^2\theta} + \frac{\beta^3}{2\cos^2\phi},$$

where $\theta = \frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}$ and $\phi = \frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}$

$$\therefore \text{L.H.S.} = \frac{\alpha^3}{1-\cos 2\theta} + \frac{\beta^3}{1+\cos 2\phi}$$

$$= \frac{\alpha^3}{1-\cos(\tan^{-1}\alpha/\beta)} + \frac{\beta^3}{1+\cos(\tan^{-1}\beta/\alpha)}$$

$$= \frac{\alpha^3}{1-\cos\left(\cos^{-1}\frac{\beta}{\sqrt{\alpha^2+\beta^2}}\right)} + \frac{\beta^3}{1+\cos\left(\cos^{-1}\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}\right)}$$

$$= \frac{\alpha^3}{1-\frac{\beta}{\sqrt{\alpha^2+\beta^2}}} + \frac{\beta^3}{1+\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}}$$

$$= \left\{\frac{\alpha^3}{\sqrt{\alpha^2+\beta^2}-\beta} + \frac{\beta^3}{\sqrt{\alpha^2+\beta^2}+\alpha}\right\}\sqrt{\alpha^2+\beta^2}$$

$$= \left[\frac{\alpha^3\left\{\sqrt{\alpha^2+\beta^2}+\beta\right\}}{\alpha^2+\beta^2-\beta^2} + \frac{\beta^3\left\{\sqrt{\alpha^2+\beta^2}-\alpha\right\}}{\alpha^2+\beta^2-\alpha^2}\right]\sqrt{\alpha^2+\beta^2}$$

$$= \left\{\alpha\left(\sqrt{\alpha^2+\beta^2}+\beta\right) + \beta\left(\sqrt{\alpha^2+\beta^2}-\alpha\right)\right\}\sqrt{\alpha^2+\beta^2}$$

$$= \alpha(\alpha^2+\beta^2) + \beta(\alpha^2+\beta^2) = (\alpha+\beta)(\alpha^2+\beta^2) = \text{R.H.S.}$$

MPP-2 MONTHLY Practice Problems

Class XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Inverse Trigonometric Functions

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. If $x_1 = 2\tan^{-1}\left(\frac{1+x}{1-x}\right), x_2 = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ where $x \in (0, 1)$, then $x_1 + x_2$ is equal to

(a) 0 (b) 2π

(c) π (d) none of these

2. The value of $\sin\{2\tan^{-1}(0.75)\}$ is equal to

(a) 0.75 (b) 1.5 (c) 0.96 (d) sin 1.5

3. If x_1, x_2, x_3 and x_4 are the roots of the equation $x^4 - x^3\sin 2\beta + x^2\cos 2\beta - x\cos\beta - \sin\beta = 0$, then $\tan^{-1}x_1 + \tan^{-1}x_2 + \tan^{-1}x_3 + \tan^{-1}x_4$ is equal to

(a) β (b) $\frac{\pi}{2} - \beta$ (c) $\pi - \beta$ (d) $-\beta$

4. If $\frac{1}{2}\sin^{-1}\left[\frac{3\sin 2\theta}{5+4\cos 2\theta}\right] = \tan^{-1}x$, then $x =$

(a) $\tan 3\theta$ (b) $3\tan\theta$

(c) $\left(\frac{1}{3}\right)\tan\theta$ (d) $3\cot\theta$

5. If $f(x) = \sin^{-1}\left(\frac{\sqrt{3}}{2}x - \frac{1}{2}\sqrt{1-x^2}\right), -\frac{1}{2} \le x \le 1$, then $f(x)$ is equal to

(a) $\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(x)$ (b) $\sin^{-1}x - \frac{\pi}{6}$

(c) $\sin^{-1}x + \frac{\pi}{2}$ (d) none of these

6. If $[\sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1}x] = 1$, where $[\cdot]$ denotes the greatest integer function, then x belongs to the interval :

(a) [tan sin cos 1, tan sin cos sin 1]

(b) (tan sin cos 1, tan sin cos sin 1)

(c) [–1, 1]

(d) [sin cos tan 1, sin cos sin tan 1]

One or More Than One Option(s) Correct Type

7. α, β and γ are angles given by $\alpha = 2\tan^{-1}(\sqrt{2}-1)$, $\beta = 3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ and $\gamma = \cos^{-1}\left(\frac{1}{3}\right)$, then

(a) $\alpha > \beta$ (b) $\beta > \gamma$

(c) $\gamma > \alpha$ (d) none of these

8. $2\tan^{-1}(-3)$ is equal to

(a) $-\cos^{-1}\left(-\frac{4}{5}\right)$ (b) $-\pi + \cos^{-1}\left(\frac{4}{5}\right)$

(c) $-\frac{\pi}{2} + \tan^{-1}\left(-\frac{4}{3}\right)$ (d) $\cot^{-1}\left(\frac{4}{3}\right)$

9. If $\sin[2\cos^{-1}\{\cot(2\tan^{-1}x)\}] = 0$, $x > 0$, then

(a) $x = 1$ (b) $x = \sqrt{2}+1$

(c) $x = \sqrt{2}-1$ (d) $x = 3$

10. If $\sin^{-1}\left(a - \frac{a^2}{3} + \frac{a^3}{9} + \ldots\right) + \cos^{-1}(1 + b + b^2 + \ldots) = \frac{\pi}{2}$, then

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(a) $b=\frac{2a-3}{3a}$ (b) $b=\frac{3a-2}{2a}$

(c) $a=\frac{3}{2-3b}$ (d) $a=\frac{2}{3-2b}$

11. If the numerical value of

$$\tan\left\{\tan^{-1}\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right\} \text{ is } \frac{a}{b}, \text{ then}$$

(a) $a+b=23$ (b) $a-b=11$

(c) $3b=a+1$ (d) $2a=3b$

12. If $x>y>z>0$, then the value of

$$\left[\cot^{-1}\frac{xy+1}{x-y}+\cot^{-1}\frac{yz+1}{y-z}+\cot^{-1}\frac{zx+1}{z-x}\right], \text{ where}$$

[.] represents the greatest integer function, is

(a) 2 (b) 3

(c) 0 (d) 4

13. Which of the following is a rational number ?

(a) $\sin\left(\tan^{-1}3+\tan^{-1}\frac{1}{3}\right)$

(b) $\cos\left(\frac{\pi}{2}-\sin^{-1}\frac{3}{4}\right)$

(c) $\log_2\left(\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)\right)$

(d) $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$

Comprehension Type

$f(x) = \sin\{\cot^{-1}(x+1)\} - \cos(\tan^{-1}x)$

$a = \cos\tan^{-1}\sin\cot^{-1}x$

$b = \cos(2\cos^{-1}x + \sin^{-1}x)$

14. If $f(x)=0$, then a^2 is equal to

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{5}{9}$ (d) $\frac{9}{5}$

15. If $a^2=\frac{26}{51}$, then b^2 is equal to

(a) $\frac{1}{25}$ (b) $\frac{24}{25}$ (c) $\frac{25}{26}$ (d) $\frac{50}{51}$

Matrix Match Type

16. Match the following:

	Column-I		Column-II
P.	If $A=\tan^{-1}\frac{x\sqrt{3}}{2\lambda-x}$ and $B=\tan^{-1}\left(\frac{2x-\lambda}{\lambda\sqrt{3}}\right)$, then the value of $A-B$ is	1.	π
Q.	If range of $f(x)=\sec^{-1}x+\text{cosec}^{-1}x+\sin^{-1}x$ is so, by then 6 equals.	2.	$\frac{\pi}{4}$
R.	$\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{3}=$	3.	$\frac{\pi}{2}$
S.	$\sin^{-1}\left(\frac{4}{5}\right)+2\tan^{-1}\left(\frac{1}{3}\right)=$	4.	$\frac{\pi}{6}$

	P	Q	R	S
(a)	1	3	4	2
(b)	2	4	3	1
(c)	4	2	3	1
(d)	4	1	2	3

Integer Answer Type

17. The number of solutions of the equation

$$\frac{(\sin^{-1}x)^3+(\cos^{-1}x)^3}{(\tan^{-1}x+\cot^{-1}x)^3}=7 \text{ is}$$

18. If $S=\sum_{r=1}^{90}\tan^{-1}\left(\frac{2r}{2+r^2+r^4}\right)$, then $\frac{8190}{8192}\cot S$ is equal to

19. If $x=\tan\left[\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right]$, then $29x$ is equal to

20. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$, then the value of

$$x^{100}+y^{100}+z^{100}-\frac{9}{x^{101}+y^{101}+z^{101}} \text{ is}$$

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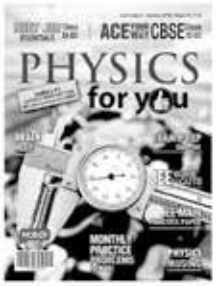

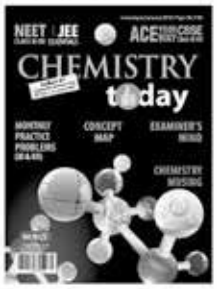

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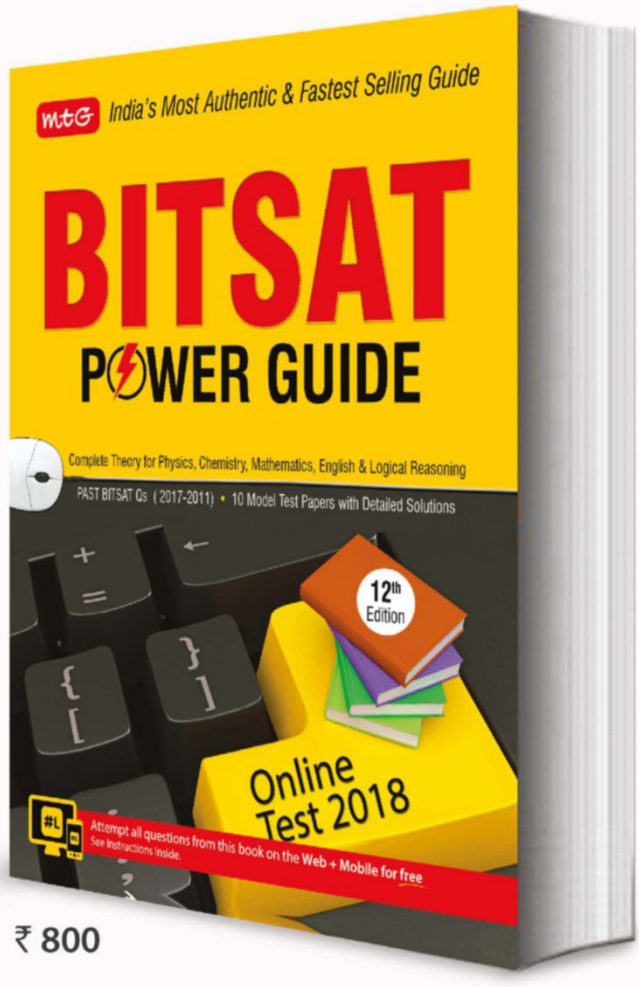
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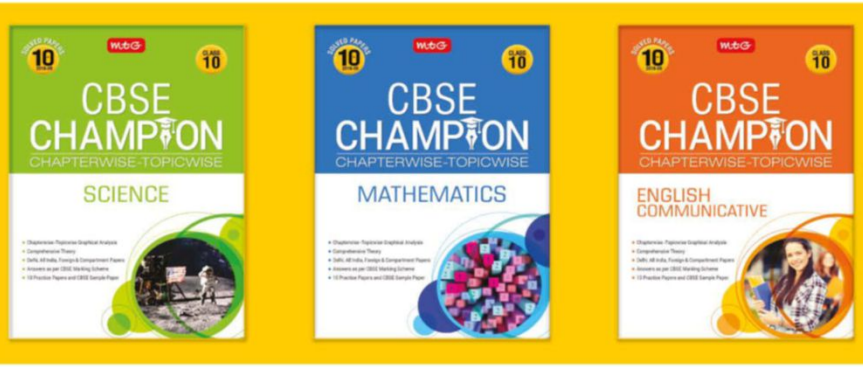

CLASS 10

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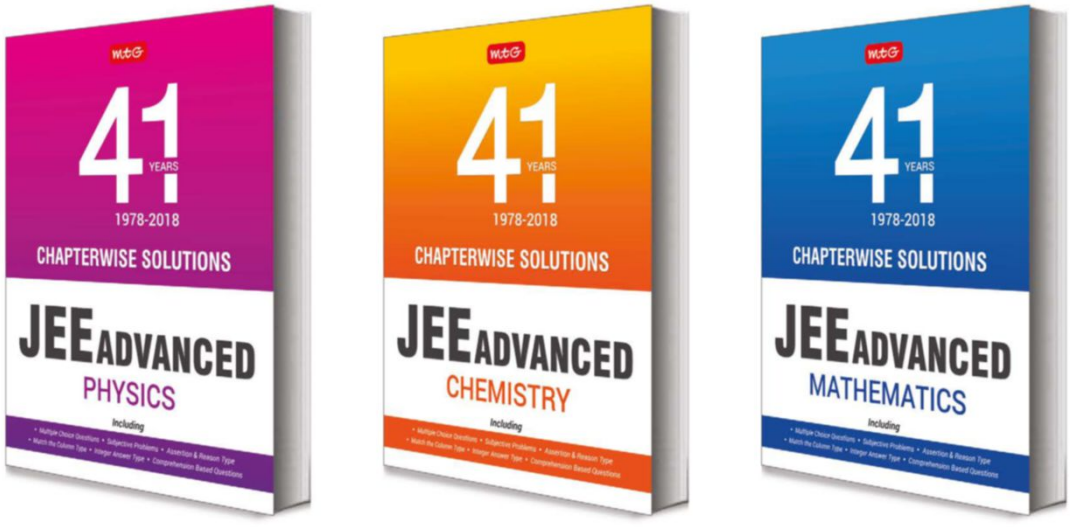

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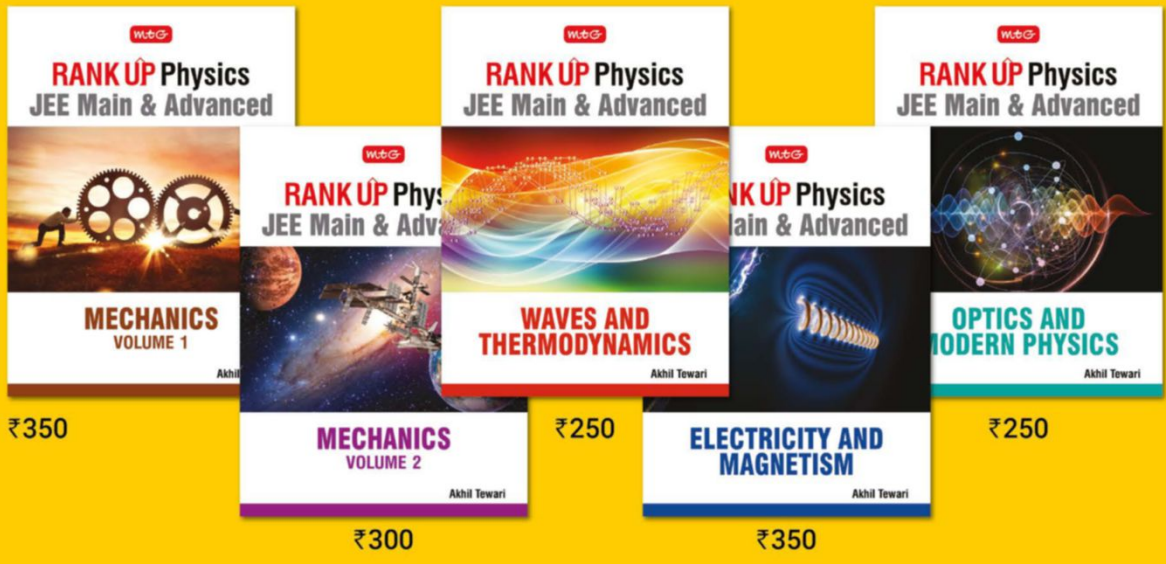

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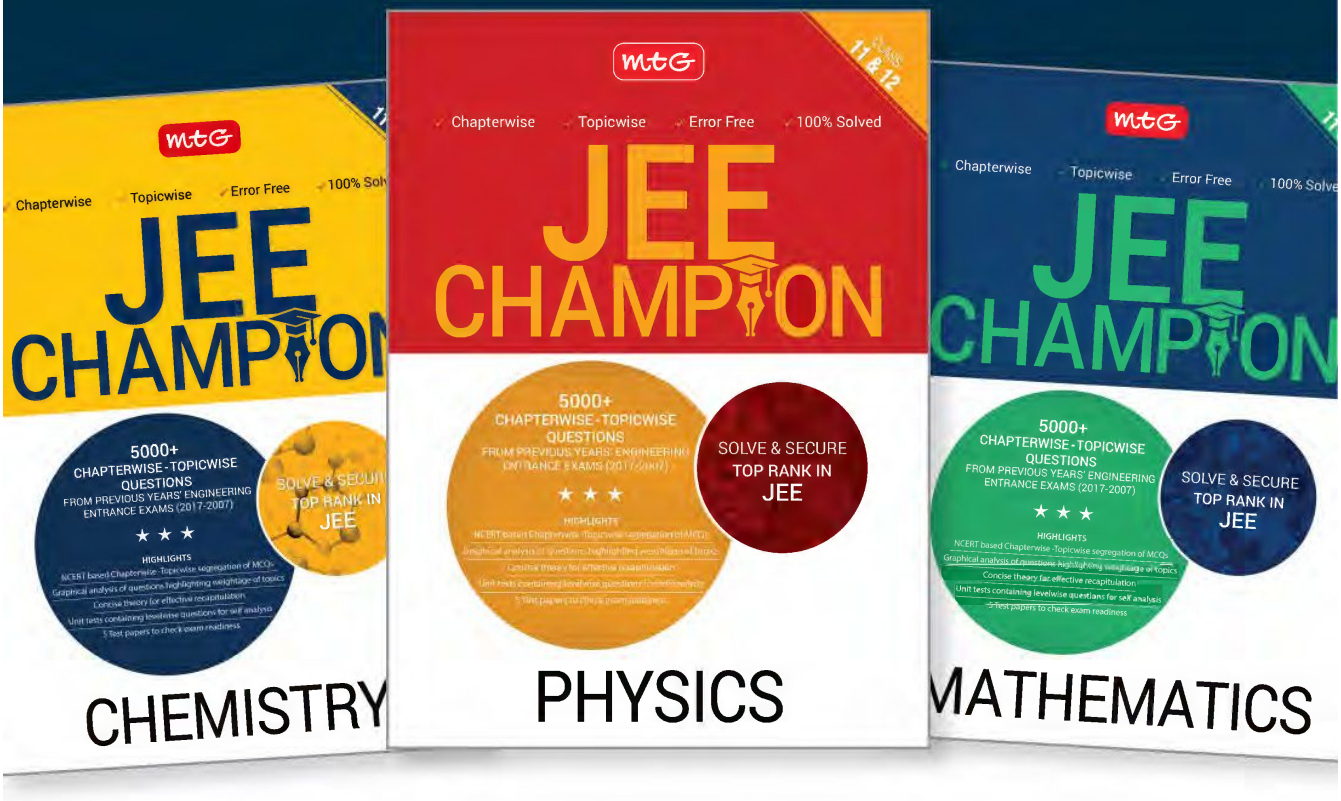

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